PARTITION AND REVISION: THE SEMANTICS 
OF COUNTERFACTUALS 

I. A STRAIGHTFORWARD ANALYSIS SEEMS TO FAIL 

What is for me the most intuitive analysis of counterfactuals goes as follows:

The truth of counterfactuals depends on everything which is 
the case in the world under consideration: in assessing them, 
we have to consider all the possibilities of adding as many 
facts to the antecedent as consistency permits. If the con-
sequent follows from every such possibility, then (and only 
then), the whole counterfactual is true.

I want to express this idea within the framework of possible-worlds seman-
tics. 

The Analysis 

Let \( W \) be the set of possible worlds. A proposition is then a 
subset of \( W \). If \( p \) is any proposition, \( p \) is true in a world \( w \) of 
\( W \) if and only if \( w \) is a member of \( p \). The notions of con-
sistency, logical compatibility and consequence are defined 
in the usual way.

Suppose now I utter a sentence of the form

\[
(1) \quad \text{If it were the case that } \alpha, \text{ then it would be the case that } \beta.
\]

where \( q \) and \( r \) are the propositions expressed by \( \alpha \) and \( \beta \) respectively. We 
have to specify how the proposition expressed by the utterance of the 
whole sentence depends on \( q \) and \( r \). Let \( p \) be this proposition. And let \( f \) be 
that function from \( W \) which assigns to every world the set of all those 
propositions which ‘are the case’ in it. Then \( p \) is the set of exactly those 
worlds \( w \) of \( W \) which meet the following condition:
If $A_w(q)$ is the set of all consistent subsets of $f(w) \cup \{q\}$ which contain $q$, then every set $X$ in $A_w(q)$ has a superset $Y$ in $A_w(q)$ such that $\bigcap Y \subseteq r$.  

If we don’t have to worry about the existence of maximal sets, we can express the same condition in the following way:

The proposition $r$ follows from every maximal set in $A_w(q)$.  
(A set is maximal in $B$ iff it has no proper superset in $B$.)

Surprisingly, this analysis seems to endow $p$ with the following truth-conditions:

**The Critical Truth-Conditions**

(i) If $q$ is true, then $p$ is true if and only if $r$ is true (in the world under consideration).

(ii) If $q$ is false, then $p$ is true (in the world under consideration), if and only if $r$ follows from $q$ (i.e., $q \subseteq r$).

The reader may be curious to see how this result is obtained. The proof is as follows.²

**The Critical Argument**

1. We assume — and this seems natural enough — that ‘what is the case’ in $w$ is to be identified with the set of propositions true in $w$.

2. Suppose $q$ is true in $w$; this means that $q \in f(w)$. Thus $f(w)$ is a superset of every subset of $f(w) \cup \{q\} - f(w)$, and clearly $r \in f(w)$ iff $\bigcap f(w) \subseteq r$. (In fact, $\bigcap f(w) = \{w\}$.) So $p$ is true in $w$ iff $r \in f(w)$.

3. Suppose $q$ is false in $w$, and let $w^* \in q$. Then $\{w^*\} = \bigcap \{-q \cup \{w^*\}, q\}$, so $\{-q \cup \{w^*\}, q\}$ is consistent. And clearly $\{-q \cup \{w^*\}, q\} \subseteq f(w) \cup \{q\}$. Thus $\{-q \cup \{w^*\}, q\} \in A_w(q)$ and hence if $w \in p$ then $\bigcap \{-q \cup \{w^*\}, q\} \subseteq r$. Hence $w^* \in r$, supposing that $w \in p$. Since this holds for arbitrary $w^*$, we have $q \subseteq r$. On the other hand, if $q \subseteq r$ clearly $p = W$ and so in particular $w \in p$.  

²