Computational Algorithms for Networks of Queues with Rejection Blocking

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Summary. Open, closed and mixed queueing networks with reversible routing, multiple job classes and rejection blocking are investigated. In rejection blocking networks blocking event occurs when upon completion of its service of a particular station's server, a job attempts to proceed to its next station. If, at that moment, its destination station is full, the job is rejected. The job goes back to the server of the source station and immediately receives a new service. This is repeated until the next station releases a job and a place becomes available. In the model jobs may change their class membership and general service time distributions depending on the job class are allowed. Two station types are considered: Either the scheduling discipline is symmetric, in which case the service time distributions are allowed to be general and dependent on the job class or the service time distributions at a station are all identical exponential distributions, in which case more general scheduling disciplines are allowed. An exact product form solution for equilibrium state probabilities is presented. Using the exact product form solution of the equilibrium state distribution, algorithms for computation of performance measures, such as mean number of jobs and throughputs, are derived. The complexity of the algorithms is discussed.

1. Introduction

Product form queueing networks (also referred to as BCMP [6] or Kelly [17, 18] or separable networks [24]) to which we will refer as classical networks in what follows, have proved invaluable in modeling a variety of computer systems, communication networks as well as manufacturing systems. They are flexible enough to represent adequately many of the features arising in such systems.
applications. They have not, however, been able to provide much insight into
the phenomenon of blocking, because all algorithms for product form networks
are based on the assumption that the stations have infinite buffer space.

In recent years there has been a growing interest in the analysis queueing
networks with blocking. These are networks where the stations have finite ca-
pacities, hence blocking can occur if a station is full to its capacity. A job which
wants to come to the full station must reside in its source station and block
it until a place is available in the destination station. The interest in networks
with blocking comes primarily from the realization that these models are useful
in the study of the behavior of subsystems of computers and communication
networks, in addition to detailed descriptions of several computer-related appli-
cations such as manufacturing systems. In recent years several investigators
have published results on networks with different blocking policies [20]. In
so-called “rejection blocking” networks the blocking event occurs when a job
completing service at station i’s server wants to join station j, whose capacity
is full. The job is rejected by station j. That job goes back to station i’s server
and receives another round of service. This activity is repeated until station
j releases a job, and a place becomes available. The “rejection blocking” type
has been used to model systems such as communication networks, computer
systems with limited multiprogramming, production lines and flexible manufac-
turing systems. Two variants of this blocking policy can be considered:

i) The destination of the job once it finishes service is fixed.

ii) The job selects a destination independently after each round of service
according to the routing probabilities.

The definition of the second variant of rejection blocking makes deadlocks
impossible if the network is irreducible (i.e., each station is reachable from every
other station). As long as there is at least one free space in some station a
job will eventually move into it, even if this takes a long sequence of trials.
This makes a general analysis much simpler. No restrictions that make deadlocks
impossible are needed and no special method to handle deadlocks has to be
included in the model.

Note that for tandem and cyclic networks there is no difference between
the variants of rejection blocking, since there is only one possible destination
for each job. Same thing happens in a merge configuration where several stations
feed a single station downstream. For networks with arbitrary topology the
two variants of rejection blocking are different.

In this paper we investigate queueing networks with rejection blocking of
variant ii). Once a job in class α finishes service in station i it determines, accord-
ing to the routing probabilities $p_{iα,jβ}$, to which station j and class β it goes
next. With a certain probability (that depends on the state of the destination
station) the job will be rejected there. The rejected job returns to station i
(in class α) to get another round of service, independent from the one it received
before. When this new round of service is finished, the job again selects a destina-
tion station and class (independent from the ones selected before) and so on.

The rejection blocking policy was introduced by Caseau and Pujolle [8]
who considered tandem networks only. They investigate various blocking poli-