1. Introduction

In any Lorentz frame of reference \((x^i, t)\), where \(x^i (i = 1, 2, 3)\) are rectangular Cartesian spatial coordinates and \(t\) is the time, \textsc{maxwell's} equations in a 

stationary, polarizable and magnetizable medium can be written in the form

\[
\frac{\partial}{\partial t} \int_{\mathcal{L}} B \cdot d\mathbf{a} + \oint_{\mathcal{L}} E \cdot dl = 0, \tag{1.1}
\]

\[
\oint_{\mathcal{L}} B \cdot d\mathbf{a} = 0, \tag{1.2}
\]

\[
\oint_{\mathcal{L}} H \cdot dl - \frac{\varepsilon}{\varepsilon_0} \int_{\mathcal{L}} D \cdot d\mathbf{a} = \int_{\mathcal{L}} J_F \cdot d\mathbf{a}, \tag{1.3}
\]

\[
\oint_{\mathcal{L}} D \cdot d\mathbf{a} = \int_{\mathcal{L}} Q_F \cdot d\mathbf{v}, \tag{1.4}
\]

\[
D = \varepsilon_0 E + P, \quad H = \mu_0^{-1} B - M, \tag{1.5}
\]

where \(B\) is the magnetic flux density, \(E\) the electric field, \(H\) the magnetic intensity, \(D\) the electric displacement, \(J_F\) the density of free current, \(Q_F\) the density of free charge, \(P\) the polarization, \(M\) the magnetization, and \(\varepsilon_0\) and \(\mu_0\) are fundamental constants whose product \(\varepsilon_0 \mu_0 = c^{-2}\) where \(c\) is the velocity of light in vacuum. \(dl\) denotes an element of an arbitrary closed contour \(\mathcal{L}\) in space and \(d\mathbf{a}\) an element of a surface \(\mathcal{A}\) bounded by \(\mathcal{L}\).

The system of equations (1.1) to (1.5) is underdetermined, and to obtain a determinate system of equations amongst the twenty-two unknown field components \((B_i, E_i, H_i, D_i, P_i, M_i, J_F, Q_F)\) it is necessary to supplement the basic Maxwell equations with certain constitutive relations. The form of these constitutive relations depends on the nature of the \textit{material medium} in which the electromagnetic field \((E, B)\) resides. The great mathematical variety of these electromagnetic constitutive relations makes possible a great variety of physical phenomena embraced by and consistent with \textsc{maxwell's} framework of equations (1.1) to (1.5).

The simplest of all media, \textit{vacuum}, is characterized by the constitutive relations

\[
J_F = Q_F = M = P = 0. \tag{1.6}
\]
The next simplest medium is the rigid, linear, stationary, non-conducting dielectric, for which the constitutive relations

\[
\mathbf{Q}_F = \mathbf{J}_F = 0,
\]

\[
\mathbf{D} = \varepsilon \cdot \mathbf{E}, \quad \mathbf{B} = \mu \cdot \mathbf{H},
\]

were given by Maxwell [1873, § 784]. Here \(\varepsilon\) and \(\mu\) are constant second order tensors, proportional to the unit tensor if the medium be isotropic. As is known, constitutive relations of this simple type do not account for the observed absorption and dispersion of electromagnetic waves in non-conductors, nor does Maxwell's device of including a linear law of conduction, \(\mathbf{J}_F = C \cdot \mathbf{E}\), replacing (1.7)_2, suffice to account for the observed magnitude of the "dielectric losses" when a material is placed in a variable electric field (Cf. Whitehead [1927, Lecture 1]).

Hopkinson [1877], following a suggestion of Maxwell, proposed a constitutive relation for the electric displacement in a dielectric having the form

\[
\mathbf{D}(t) = \varepsilon \mathbf{E}(t) + \int_{-\infty}^{t} q(t-\tau) \mathbf{E}(\tau) d\tau,
\]

where the function \(q(u), u \geq 0\), is a decreasing function of \(u\). By suitable adjustment of the memory function \(q\), Hopkinson was able to correlate his data on the residual charge of Leyden jars. In proposing (1.8), Hopkinson was guided by the earlier mechanical constitutive relation of Boltzmann [1874] in which the torque and twist of a wire are related by a formula identical with (1.8).

Maxwell (cf. Rayleigh [1899]; Whittaker [1951]) initiated also the method which enjoys current favor of deriving electromagnetic constitutive relations from a molecular model of a material medium and the laws of mechanics. Elaborate theories of electromagnetic absorption and dispersion of this general type were developed by Drude [1893, 1902], Voigt [1899], and many others. A detailed and modern theory of electromagnetic constitutive relations as derived from the dynamics of an ionic crystal lattice, including an appropriate application of quantum mechanics, is given in the book by Born & Huang [1954].

Volterra [1912] extended the Hopkinson's relation (1.8) to the nonlinear, anisotropic, and magnetic case. Volterra's most general expression of the idea took the form of the equations:

\[
\mathbf{D}(x, t) = \varepsilon \cdot \mathbf{E}(x, t) + F[\mathbf{E}(x, \tau)],
\]

\[
\mathbf{B}(x, t) = \mu \cdot \mathbf{H}(x, t) + \Phi[\mathbf{H}(x, \tau)],
\]

where (1.9)_1 reduces to (1.8) if the functional \(F\) is linear, isotropic, and satisfies certain other conditions of which we shall say more later. Volterra's theory was developed further by Graffi [1927, 1928].

Volterra's paper, though it treats the general anisotropic case and expresses a very general view, makes the traditional a priori separation of electric and magnetic effects characteristic of Maxwell's simple relations (1.7). That this separation is inadequate to account for numerous known phenomena such as optical activity and the rotation of the plane of polarization of light by a strong magnetic field (Faraday effect) should have been apparent from the earlier