Elasto-plastic dynamic analysis of plane frames and deep arches

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Abstract. The dynamic response of elasto-plastic frames and arches is investigated using a discrete system approach. The governing equations of motion are formulated through the virtual work principle and supplemented by the compatibility conditions established through the conjugate segment analogy. Time marching is carried out through direct time integration process using backward differences. The method, requiring small core storage and short computer time, can be easily implemented on any personal computer.

1 Introduction

The dynamic analysis of elasto-plastic structures is often required in the design consideration of structures subjected to shortlived but severe loadings, e.g., the disaster prevention design of buildings for earthquake and blast loadings. The subject has been studied by several researchers in the past few decades. Among others, Heidebrecht, Fleming and Lee (1963) adopted a discrete system approach to study the response of elasto-plastic beams. The plastic hinges were assumed to occur only at various chosen points of concentrated masses. Either $M_i$, the bending moment at the mass point $i$, if that part remained elastic or $\phi_i$, the plastic hinge rotation was chosen as the variable at that point. The forced vibration of elastic planar curved beams was studied by Tene, Epstein and Sheinman (1975). The effect of viscous damping was later considered by Sheinman (1979). Backward differences were adopted to solve the governing equations in both space and time domains. Displacement components were chosen as basic variables in their studies.

In the present paper, the discrete system approach proposed by Heidebrecht, Fleming and Lee (1963) is extended to study the dynamic response of elastic-perfectly plastic arches. The governing equations are formulated through the virtual work principle and supplemented by the compatibility conditions established through the conjugate segment analogy proposed by Lee (1958). Direct time integration is adopted for the time marching scheme using backward differences suggested by Houbolt (1950).

2 Assumptions and structural idealization

The following assumptions are adopted. The loadings are assumed to be in the plane of the structure. An elastic-perfectly plastic moment-curvature relationship is assumed. The effect of axial deformation is neglected but the effect of axial force on the plastic moment is considered.

The arch is discretized into $(k - 1)$ straight elements with $k$ nodes as shown in Fig. 1. The loadings, masses and damping effects are lumped and the plastic hinges are allowed to form only at the nodes. The bending moment $M_i$ and the plastic hinge rotation $\phi_i$ are chosen as variables at node $i$. When the node responds elastically, $\phi_i$ remains constant whereas if actual or plastic hinge exists, $M_i$ remains zero or assumes the value of the plastic moment corresponding to the axial force on the section respectively. Thus only one unknown per node has to be dealt with at any given time.
3 Equations of motion

The D’Alembert forces at any node $i$, $Q_{xi}$ and $Q_{yi}$ in Fig. 2, can be expressed as the resultants of the applied loads, the inertia forces and the damping resistance in the form

\[ Q_{xi} = p_{xi} - m_{xi} \ddot{u}_i - c_{xi} \dot{u}_i \]
\[ Q_{yi} = p_{yi} - m_{yi} \ddot{v}_i - c_{yi} \dot{v}_i \]  

(1)

where $p_{xi}$ and $p_{yi}$ are the applied loads, $m_{xi}$ and $m_{yi}$ the lumped masses, $c_{xi}$ and $c_{yi}$ the damping coefficients and $u_i$ and $v_i$ the displacement components respectively, superdots denoting differentiation with respect to time. Equation (1) can be expressed in matrix form as

\[ Q = p - m \ddot{u} - c \dot{u} \]  

(2)

Consider the mechanism, i.e., the virtual displacement of a segment of any three consecutive elements as shown in Fig. 2. Applying the principle of virtual work yields

\[ -e_0 M_i + e_1 M_{i+1} - e_2 M_{i+2} + e_3 M_{i+3} = -a_{x1} Q_{x(i+1)} + a_{y1} Q_{y(i+1)} + a_{x2} Q_{x(i+2)} - a_{y2} Q_{y(i+2)} \]  

(3)

where $M_i$ is the bending moment and $e_i$, $a_{xi}$ and $a_{yi}$ the functions of coordinate of node $i$ measured from the origin which may be arbitrarily located.

There are $(k - 3)$ such independent mechanisms for an arch yielding $(k - 3)$ equations of motion, which can be expressed in collective form as

\[ E \cdot M = A \cdot Q \]  

(4)

When $s$ adjacent elements lie on the same straight line, $(s - 1)$ equilibrium equations can be derived from $(s - 1)$ beam mechanisms involving only two adjacent elements. Another independent equilibrium equation is derived from the sway mechanism consisting of the straight segment and the preceding and trailing elements. The same formulation described earlier still applies.

Since coefficient matrices $A$ and $E$ are sparse and usually well organized, only non-zero entries are registered and dealt with. This saves substantially the core storage requirement and the computing time. An index vector is, however, necessary to locate the positions of these non-zero entries when a sway mechanism with more than one middle element is involved.