The influence of geometry on the ascent of magma in open fissures

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Abstract. During steady eruption, the flow conditions (emitted mass flux, exit velocity and exit pressure) depend on the geometry of the conduit in which the eruption occurs. This dependence is examined for the one-dimensional, isothermal ascent of a homogeneous basaltic magma with an aqueous volatile phase and newtonian rheology. By fixing the geometry of the feeding fissure, the mass flux flowing in steady conditions can be determined at any depth, as well the magma pressure and vertical velocity. Flow behaviour is analysed for three fissure shapes: constant width, slowly upward narrowing and lenticular. In all the cases examined the magma arrives at the earth's surface with a pressure greater than atmospheric. The results are compared with those obtained when a lithostatic pressure gradient is assumed for the magma column. Some speculations are made, moreover, about the change in eruption style, if conduit geometry varies during a non-steady phase.

Introduction

Several models have been proposed to describe the ascent of magma in an open conduit (Pai et al. 1978; Kieffer 1977; Wilson et al. 1980; Wilson and Head 1981; Vergniolle and Jaupart 1986; Buresti and Casarosa 1989). The objective of such studies is to relate the dynamical variables of eruption (fluid pressure, ascent velocity, emitted mass flux) to the properties of the magmatic fluid (viscosity, density, composition and volatile content) and to the geometrical characteristics of the conduit (shape, source depth and so on).

The subject has been extensively studied in the works of Wilson et al. (1980), and Wilson and Head (1981), where the flux equations have been solved, numerically, under the hypotheses of steady, isothermal, unidimensional flow for a homogeneous two-phase newtonian fluid. A special emphasis has been given there to the characterization of the motion when the pressure variation with depth during the ascent was supposedly known and, in particular, equal to the lithostatic (hydrostatic) gradient. The exit pressure at the surface was consequently atmospheric and the equations of motion were solved to determine the shape of the conduit and the ascent velocity. The typical shape one obtains is of a conduit with an almost constant section up to the depth where the (local) sonic velocity of the liquid is reached, and then a widening of the section up to the earth's surface.

It is not clear, however, that the pressure gradient in a magma flow has to be lithostatic. There are estimates based on the dynamics of historical eruptions (Kieffer 1981) and rheological arguments (Pinkerton and Sparks 1978) suggesting that, at the exit on the earth's surface, the pressure is greater than the atmospheric pressure. Moreover, the assumption of a lithostatic pressure gradient suggests the possibility of elastic deformations of the walls of the conduit, whereas it is plausible either that the walls are sufficiently rigid to substatn stresses or that they react to the pressure excess by just fracturing, especially if a conduit is opened for tectonic reasons and not from the intrusion of the fluid.

In this paper we extend the analysis of Wilson et al. (1980) and Wilson and Head (1981) by considering flow in open fissures of known geometry. The steady solutions of the problem are obtained by solving numerically the non-linear motion equations, taking into account that the exit values of the pressure and velocities must be consistent with physical conditions that ensure steady flow, and that the presence of a singularity in the motion equations restrict the number of possible solutions in fixed shape fissures. Since the exit values depend on the initial conditions and input data, the steady solutions have been found out by varying the initial data until all the physical conditions were satisfied.

In the subsequent sections we will discuss in detail the assumptions of the model and the physical conditions to satisfy to obtain steady motion. The motion equations are then solved for different fissure shape and a short analysis on the role played by variations in
viscosity, gas content and initial pressure at depth in changing the exit dynamical variables will be presented. Finally, some comments are made on the possible changes in flux conditions when changes in geometry occur, considering this unsteady process as a sequence of successive, slightly different, steady processes.

Magma-ascent model

The flow of magma from a high pressure region (i.e. magma reservoir) to the earth's surface through an open fissure is described schematically as follows. At depth, where the pressure is sufficiently high to prevent the volatile exsolution, the magma flows essentially as a viscous incompressible fluid. Progressive decompression favours the exsolution of the volatile phase in bubbles, whose shapes and dimensions depend on the rheological properties of the gas and of the entrapping liquid, on the relative ascent velocity of the bubbles with respect to the liquid, and on the gas solubility and diffusivity. When the bubbles occupy about three-quarters of the total volume the foam fragments in a mixture of gas and pyroclasts of various dimensions (Sparks 1978).

In modelling this process we make the following assumptions:

a) Homogeneous flow. This assumption requires a continuous mechanical equilibrium among the coexisting phases and, consequently, a common ascent speed. As shown by Vergniolle and Jaupart (1986) such a condition is plausible for high viscosity magmas.

b) Isothermal flow. The temperature will remain constant during ascent if there is an efficient heat exchange among the various phases of the mixture. The efficiency can be questionable when the mixture is composed of gas and solid pyroclasts. Buresti and Casarosa (1989) have shown by studying the adiabatic flow of gas-particle mixtures that the temperature decreases are negligible when the flow occurs in a conduit of almost constant section. Since our analysis is essentially restricted to straight fissures, or fissures with very small changes in slope, we will maintain the assumption of isothermcity. Moreover, we assume that there is no substantial heat loss due to conduction at the conduit walls, nor heat gain due to gas exsolution or friction during the motion. Again, these approximations can be justified over a wide range of conditions (Wilson et al. 1980).

c) One-dimensional flow. Assuming that fissures have horizontal and vertical lengths several orders of magnitude larger than their widths (kilometers compared to a few meters), we can neglect edge effects. Since, moreover, we consider flows in fissures with almost constant sections, only the vertical changes in the dynamical variables will be significant (one-dimensional motion). Strictly, the dynamical variables (pressure, velocity and density) will exhibit horizontal variations due to friction force, so their values in the equations of motions must be considered as values averaged across horizontal sections.

d) Newtonian rheology. The viscosity of the fluid is assumed to be constant during the ascent. This is the least justifiable of our assumptions, since it is known that magma viscosity changes appreciably during gas exsolution (Shaw 1965). This effect will not be taken into account in this paper, but will be included in subsequent analyses now in preparation.

e) Steady motion. The quasi-stationarity of many volcanic eruptions is not in doubt, the time required for magma to rise from the source region being much less than the duration of the eruption (hours in respect to a few minutes).

Equations of motion

We study the flow in fissures of horizontal length \( L \) and half-width \( w \). The reference system for the vertical coordinate \( h \) is upwards, with the origin at the depth at which we start to consider the motion. The earth’s surface is at \( h_0 \). The fluid has temperature \( T \), global density \( \rho \) and viscosity \( \eta \) and contains a weight fraction \( n_0 \) of volatile phase.

The motion is described by the following equations:

a) Mass conservation:

\[
m = 2 \rho_0 w = \text{constant} \tag{1}
\]

where \( m \) is the mass flux flowing per horizontal length and \( u \) is the vertical, horizontally averaged, velocity of the fluid.

b) Energy conservation:

\[
dP \over dh = \left( g + \frac{1}{2} \frac{f a^2}{r_H} \right) \left( dh + \alpha u du \right) \tag{2}
\]

where \( P \) is the pressure in the fluid, \( g \) is the gravity acceleration, \( f \) is the Fanning friction factor, \( r_H \) is the hydraulic radius of the fissure and \( \alpha \) is correction factor. The viscosity is contained in the Fanning friction factor, which depends on the Reynolds number \( Re = 4 \rho_0 ur_H / \eta \) and on the roughness of the walls (Schlichting 1968) through the relation:

\[
f = \frac{24}{Re} + f_0
\]

where \( f_0 \) is a constant such that \( f \approx f_0 \) for \( Re > 3400 \) (turbulent motion). The hydraulic radius \( r_H \) is defined as the ratio of the section area to the wetted perimeter; for a long, \( (L \gg w) \), fully filled fissure \( r_H \) is almost equal to the fissure half-width \( w \). The corrective term \( \alpha \) has been introduced to account for the fact that the mean kinetic energy does not coincide with the kinetic energy evaluated by the horizontal averaged velocity \( u \). The value of \( \alpha \) depends on the motion regime and magma rheology and can be evaluated assuming appropriate profiles for the horizontal velocity.

c) Equation of state

Gas solubility in a magmatic liquid may be expressed as a function of the pressure (Wilson et al. 1980) by:

\[
n_0 = \sqrt[3]{P
\]