CONCRETE CAN BE ABSTRACT: A CASE STUDY

ABSTRACT. This paper discusses the relationship between arithmetical concepts and solution procedures or methods. A case study is presented to illustrate the contention that some children with relatively sophisticated concepts express those concepts by using primitive methods. The process of distinguishing between these children and those with primitive concepts involves inferring the children's purposes, intentions, and anticipations about the results of their potential problem solving activity. The case study of the child’s learning in the context of direct instruction also indicates that her awareness of the limitations of her current methods played a crucial role in her development. The generality of this analysis is then illustrated by drawing analogies from the philosophy of science and mathematics, and from an analysis of the activity of professional practitioners.

There seems to be a consensus among many constructivist and information-processing early number researchers that improvements in arithmetical problem solving performance are primarily due to advances in conceptual knowledge. Riley et al. (1983) found, for example, that children who used a particular counting method to solve one semantic type of word problem frequently failed when presented with other types of arithmetical word problems that could be solved by using the same counting method. They accounted for this observation by suggesting that children at different performance levels construct differing problem representations when they interpret a particular type of task.

Steffe et al.’s (1983) constructivist analysis of counting solutions also focused on the conceptual knowledge that children use to give meaning to tasks. For these researchers, a solution procedure is viewed as an attempt to express or fill out a conceptual structure in a particular situation. As a conceptual structure can be expressed in a variety of different ways, children with structurally similar concepts of addition or subtraction might use different methods when they attempt to solve the same task. In particular, some children with relatively sophisticated concepts might routinely produce perceptually based solutions in which they establish collections of visible items. There might, therefore, be variations not only in the conceptual structures that children establish but in the ways that children express similar structures in particular situations.

This possibility, which is also supported by the findings of Baroody and Gannon’s (1984) investigation of commutivity, challenges analyses in which early number development is characterized primarily in terms of...
changes in children's solution methods. For example, children who are at Carpenter and Moser's (1984) second level and who "could solve problems only by modelling them with physical objects" (p. 179) might have a variety of qualitatively distinct concepts. The problem solving behaviour of Melissa, a child who was completing her first grade year, lends credence to this contention. Melissa's case study is also of interest because it illustrates the crucial role that problematic situations can play in precipitating the child's construction of new methods. As will be seen, however, what counts as a problem is relative to the child's current ways of operating. Two sessions were conducted with Melissa, the first a 20 minute clinical interview and the second a teaching session that lasted 45 minutes. My purpose in the two sessions, both of which were video-taped for later analysis, was to investigate Melissa's methods for finding sums, missing addends, and differences, and thus to infer the meanings she established in arithmetical situations.

THE CLINICAL INTERVIEW

Observations

As the interview progressed, it became apparent that Melissa's arithmetical solutions were based on finger patterns that could include both perceived and visually represented fingers. Consider, for example, her solution to a missing addend task in which felt squares were hidden beneath two cloths. She was told that nine squares were beneath one cloth, there were thirteen in all, and asked to find how many were screened by the second cloth. Melissa first put up nine fingers simultaneously as she said, "Nine." She then pointed to her remaining digit, the thumb of her right hand, and to three locations to the right of her thumb while she synchronously whispered "10, 11, 12, 13." Finally, she gave four as her answer.

Melissa solved this task by counting beyond a finger pattern associated with "nine" until she established a finger pattern for "thirteen". Her answer indicates that she maintained a separation between the initial finger pattern and the perceptual and represented items she subsequently counted. She also produced analogous solutions to subtraction tasks. On one occasion, she was asked to find how many marbles were left in a cup given that there were initially fourteen and eleven had been removed. She put up all ten fingers simultaneously as she said, "Fourteen." Next, she pointed to four locations to the right of her right hand while whispering "1, 2, 3, 4" and continued "5, 6, . . . , 11" as she sequentially