Abstract. Sixty-one children, from 4 to 11 years old, were presented with two sets, each containing blue and yellow elements. Each time, one colour was pointed out as the payoff colour (POC). The child had to choose the set from which he or she would draw at random a POC element in order to be rewarded. The sets were of varying sizes with different proportions of the two colours. The problem was to select the higher of the two probabilities. Three kinds of materials were used: Pairs of urns with blue and yellow beads, pairs of roulettes divided into blue and yellow sectors, and pairs of spinning tops, likewise divided into two colours.

Roughly around the age of six, children started to select the greater of the two probabilities systematically. The dominant error was selecting the set with the greater number of POC elements. Verbal concepts of probability and chance were explored and some 'egocentric' thought processes were described. The study indicates that probability concepts could be introduced into school teaching even in the first grades. The deterministic orientation in the instruction for young ages should be attenuated, permitting concepts of uncertainty right from the beginning.

1. INTRODUCTION

The beginning of children's understanding of probability, and the development of that understanding, were studied by various investigators (Piaget and Inhelder, 1951; Yost, Siegel and Andrews, 1962; Davies, 1965; Goldberg, 1966; Carlson, 1970; Fischbein et al., 1970; Hoemann and Ross, 1971; Chapman, 1975). A comprehensive review of the subject can be found in Fischbein (1975, Chapter VI).

A decision-making technique, involving two sets of elements, was used in all the studies. The elements were one of two colours. In each task, one colour was pointed out as the payoff colour (POC). The child had to choose the set from which he would try to draw at random an element of the required colour. Success in drawing an element of the POC granted him a prize. In order to be maximally rewarded, his problem was to identify the set with the higher probability of the desired colour.

Although the different studies all presented paired comparisons of binary sets, serious disparities were obtained in the investigators' conclusions concerning the age of attainment of the probability concept (Fischbein, 1975). One could hypothesize that the contradictory results stemmed from differences in the mathematical features of the problems presented. An analysis of the problems used in the studies indicates that some investigators included certain components in their comparisons that the others overlooked.
An example could clarify the above conjecture: Yost et al. (1962) presented paired comparisons only between complementary probabilities (and so did Davies, 1965), e.g., 1/4 was compared with 3/4, 1/5 was compared with 4/5, etc. Furthermore, their comparisons always included an equal number of chips of one of the colours (the colour more preferred by the child) in both boxes. Imagine a task in which both the right and left boxes contain 3 red chips, while in addition, the left box contains one blue chip and the right box 9 blue chips. Suppose blue is pointed out as the payoff colour. The child can easily decide to choose the right box just by comparing the absolute numbers of blue chips in the two boxes (9 > 1). Even if red were chosen as the POC for the same setting of boxes, the child could 'get away' with comparing two absolute quantities (consider the number of blue chips: 1 < 9, therefore choose the left box). Two unequal complementary probabilities are always on different sides of the fraction 1/2, which simplifies the task even more because it enables absolute comparison within each box. No wonder that Yost et al. (1962), Davies (1965) (and also Goldberg, 1966) found that preschool children (even 4-year-olds) performed significantly above chance level and hence concluded that these children were capable of systematic probability judgments. Note the limitations of the comparisons described above: One component of the concept 'probability of the payoff colour' in a given box is 'the number of elements of the payoff colour' in that box. This number was never smaller in the correct box than in the other; it was either equal to or greater than the number of payoff elements in the other box. In contrast to Yost et al., Piaget and Inhelder (1951) and Fischbein et al. (1970) claimed that systematic understanding of probability does not appear until a much later age. According to Piaget and Inhelder, children begin to give correct solutions at an age of 9–12 years, and even then, problems are solved by intuitive methods rather than on the basis of formal reasoning. The conclusion of Piaget and Inhelder is that fundamental probabilistic notions are not constructed until the level of formal operations because the concept of proportion requires 'operations on operations'. According to Fischbein et al., the mental mechanisms necessary for the active understanding of proportionality are already present at the level of concrete operations (9–10 years) but must be brought into play by means of brief instruction. Examination of the structure of the problems used in the latter studies reveals that the child had no simple way to solve the problems by merely comparing absolute quantities. The use of problems where the correct choice included fewer elements of the POC appears to account for the finding of an older age in which the understanding of probability emerges. Indeed, Piaget and Inhelder (1951) found that children can solve problems of the type used by Yost et al. a lot earlier.