CHOICE OF OPERATION
IN VERBAL ARITHMETIC PROBLEMS:
THE EFFECTS OF NUMBER SIZE,
PROBLEM STRUCTURE AND CONTEXT

ABSTRACT. 12- and 13-year-olds were tested with two types of tasks to test their understanding of applications of the multiplication and division of positive numbers: (i) writing down calculations required to solve verbal problems, and (ii) making up stories to fit given calculations. Selected pupils were interviewed to investigate further the thinking processes involved. The results indicate (a) the pervasive nature of certain numerical misconceptions, (b) the effects of structural differences among the items; particularly whether multiplication can be conceived as repeated addition or not, and whether division has the structure of partition, quotition or rate, (c) specific effects of context attributable to such aspects as relative familiarity, and (d) various interactions between these three sets of factors.

INTRODUCTION

A considerable body of work now exists on children's understanding of addition and subtraction (Carpenter et al., 1981); rather less, but still a significant amount, of research has been done on multiplication and division (Vergnaud, 1979; Rouchier, 1980; Brousseau, 1981). One well-known phenomenon is that when pupils are asked to choose the appropriate operations for verbal problems, errors often arise from misconceptions such as multiplication always makes bigger, division always makes smaller, and that division must be of a larger number by a smaller (Bell et al., 1981; Hart, 1981). Of the many aspects which require investigation, we chose to focus on the extent to which problem structure and context affect the difficulty of choice of operation with positive numbers, and how these factors interact with the effects of the numerical misconceptions.

These misconceptions arise, of course, as generalizations which are correct within a limited experience – that is, from extensive work in primary school, where division is nearly always of a larger integer by a smaller one, with a remainder being stated (if there is one) rather than the division being continued into fractions or decimals. Where multiplication by numbers or measures less than 1 occurs, it is usually in the context of learning the algorithm for multiplying fractions, and the relative size of the numbers multiplied, and the result, is not a focus of attention. The use of the calculator, because of its facility with numbers of any size, makes it natural to consider a wider range of numbers

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earlier in the primary school, and, by removing the burden of calculation, allows more attention to be given to the size relationships.

Calculator experience can thus help to eradicate the misconceptions, and as its use in the primary school becomes more extensive, it should diminish the tendency for them to arise. However, this still leaves the question of what meanings the pupils can attach to the operations. For $6 \times 4$, it is easy to think of 6 lots of 4 objects (repeated addition), and this can be extended to include repeated addition of measures (e.g. 6 lengths of string each 4.1 metres long). To find a situation modelled by $6.2 \times 4.1$, however, one has to go to a conceptually very different context, such as the area of a rectangle, unit price $\times$ quantity, speed $\times$ time, or enlargement. While $24 \div 6$ can be conceived as a partition (24 apples among 6 people, say), $6 \div 24$ cannot be so conceived unless the concept of partition is extended to include fractional division of the apples. A calculation such as $6 \div 0.5$ cannot be conceived as a partition, but can be seen as a quotition (e.g. 6 metres cut into half-metre lengths). For $5 \div 7.5$ to be interpreted as quotition, that concept needs to be extended (e.g. a man is making a rope 7.5 metres long and has completed 5 metres: how much of it has he made?).

A more complex group of situations involving multiplication and division we will refer to collectively as *rate* structures. These include questions relating to speed, price, miles per gallon, etc. Other applications generally encountered in school are rectangular area, measure conversion, and enlargement. In classifying these, we emphasize differences in structure (particularly repeated addition versus the rest for multiplication, and partition, quotition and rate for division), and differences in context. This approach contrasts with Vergnaud's (1980) more general and formal distinction between 'isomorphism of measures' and 'product of measures'. For example, it is to be expected that two situations which are equivalent in mathematical terms may differ in difficulty because of differential familiarity.

In most contexts, there are two distinct types of division problem. For speed, on the one hand there is the defining relationship speed = distance/time, and on the other, time = distance/speed. The former, which is conceptually related to partition, we will call type P; the latter, which is related to quotition, we will call type Q. In different contexts, the relative importance and frequency of occurrence of the three interrelated calculations differ, in terms of the problems encountered both in school and in real life. For example, with measure conversion the problem is generally to find one measure from the other (by multiplication or division, depending on how the conversion factor is stated); the conversion factor is normally known, so its calculation, given equivalent measures, is rarely needed. For rectangular area, the multiplication