A CONSTRUCTIVIST APPROACH TO NUMERATION IN PRIMARY SCHOOL: RESULTS OF A THREE YEAR INTERVENTION WITH THE SAME GROUP OF CHILDREN

ABSTRACT. This article is a follow-up to an earlier article (Bednarz and Janvier, 1982) which presented the results of a research study on the understanding of numeration by primary school children. That study pointed out the main difficulties children meet in learning numeration, presented a theoretical framework that made explicit a conception of numeration different from the one considered in current mathematics teaching, and also proposed a reference framework utilizable for learning and evaluating this notion. The experimentation in a classroom announced at the end of the article, was undertaken from 1980 to 1983 with the same group of children from the time they were in first grade (6-7 years old) to the third grade (8-9 years). The theoretical and reference frameworks developed in the former research study proved to be effective for developing a constructivist approach leading children to build a meaningful and efficient symbolism of number.

This article is mainly devoted to presenting the results of this three year longitudinal study (part C). At first, we shall characterize briefly our intervention based on a constructivist approach of numeration (part A). Also we shall describe the conditions under which the experimentation was carried out, and the means used to evaluate the impact on the pupils (part B).

INTRODUCTION

In the previous research project on numeration, we aimed at answering several questions we considered essential in order to diagnose children's understanding of numeration. In order to have an over-view of our conception of numeration underlying these questions, we shall restate some of the questions presented in the first article.

Do children see that conventional writing is related to collections of elements "structured in groupings"? (collections reorganized to make subsets appear that have the same number of elements).

Do children use grouping as a strategy to communicate information or operate on a collection, when that proves to be an efficient strategy?

What meaning do children attribute to representations of numbers that are submitted to them?

Can children carry out operations involving doing and undoing groupings?

Do concrete materials offer any support to children so that they can build on them and refer to them, particularly when they have to operate on collections (addition, subtraction, sharing, comparison...)

Can children work with two groupings of different order at the same time?

The analysis of answers to items related to each question pointed out

several misconceptions and difficulties developed by children in the current teaching of numeration. We summarize these here:—

— For most children, a number is an alignment of digits. Words like hundreds, tens, units are not taken at all into consideration, or they are associated with *a fragmentation, an order* of writing. This leads us to conclude that few children give a true interpretation of the digit position in terms of groupings.

— Many children *don’t see the usefulness (relevance) of grouping* in a task where that proves to be an efficient strategy. Some children form groupings but only in order to count the collection (by ten, by five, . . .). In these cases, we could observe that the task asked was not performed by reflecting on the groupings they made, but rather they applied learned algorithms or mental calculation facts. Few children see the validity of doing groupings to perform a task as a whole, and see that writing is a code which springs directly from these groupings.

— Few children can really operate on groupings when they have to do or undo them. Rather they try to re-transcribe with conventional writing what they do in the algorithmic procedure. We observed that children cannot illustrate or explain with any material the operations performed with conventional number writing. We find then different *erroneous interpretations of carrying and borrowing* in operations.

— Few children *can work simultaneously* in a task with *groupings of different order*. Even if they see groupings, which is not always the case, children have difficulty in co-ordinating them. In their reasoning, they confound the two different groupings.

We could pursue the analysis of children’s answers but their understanding is well enough characterized to convince us that the skills developed by school mathematics teaching do not transfer to any of the numeration tasks proposed to them. In fact a lot of children’s misconceptions can even be explained by analyzing the perception of numeration found in school text books, curriculum . . . . This point is discussed more fully in Bednarz and Janvier (1984).

By reading over the previous questions it may be striking to note the insistence we put on collections structured in groupings and on representations of numbers. Indeed for us the symbolic conventional writing is not questioned in itself, but in regard to its meaning in terms of groupings and in terms of the transformations made on these groupings when we operate. When we work on numeration, we work on the process of representation of number. The teaching of numeration should promote this process and take into consideration the operations on collections reorganized in groupings.