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THE CONCEPT OF CHANCE IN EVERYDAY TEACHING: ASPECTS OF A SOCIAL EPISTEMOLOGY OF MATHEMATICAL KNOWLEDGE

ABSTRACT. The paper analyzes the relationship between the epistemological nature of mathematical knowledge and its socially constituted meaning in classroom interaction. Epistemological investigation of basic concepts of elementary probability reveals the theoretical nature of mathematical concepts: The meaning of concepts cannot be deduced from more basic concepts; meaning depends in a self-referent manner on the concept itself. The self-referent nature of mathematical knowledge is in conflict with the linear procedures of teaching. The micro-analysis of a short teaching episode on the concept of chance illustrates this conflict. The interaction between teacher and students in everyday teaching produces a school-specific understanding of the epistemological status of mathematical concepts: the concept of chance is conceived of as a concrete generalization, which takes "chance" as a fixed and universalized pattern of explanation instead of unfolding potential and variable conceptual relations of "chance" or "randomness" and developing the theoretical nature of this concept in an appropriate way for students' comprehension.

1. INTRODUCTION

In analyses of everyday mathematics instruction, the role of two opposed interpretations of the character of mathematical knowledge is becoming increasingly important. Some consider mathematical knowledge to be objective, while others stress the subjective social conditions of the classroom interaction in which mathematical knowledge is constituted. This paper presents a research study on the teaching of probability, part of broader research on interaction in the mathematical classroom. Whereas much research done in this field takes a constructivist perspective or is based on theories of communication, we will focus primarily on the epistemological constraints on mathematical knowledge in student-teacher interactions. Our specific interest will be to understand better how in everyday teaching, processes of concept development are organized and how the meaning of mathematical concepts is constituted through social interaction.

This paper attempts to elaborate connections and differences between objective and subjective elements of mathematical knowledge by means of an epistemological conception and a case study analysis of a classroom episode. In the first part, the underlying epistemological perspective is explained using the concepts of probability and randomness as examples. The second part relates this knowledge frame to the curriculum of proba-

bility as it is used in everyday teaching. Finally, the third part presents a
detailed analysis of a short teaching episode on introducing the concept of
chance and discusses some interactively constituted epistemological pat-
terns of stochastical knowledge.

At first sight there would seem to be a wide gap between a historical,
epistemological sketch of the development of theoretical knowledge and a
case study of a short interactive teaching episode. The objective of this
investigation is not to construct a logical, deductive connection between the
historical and the interactive constitution of knowledge, however, but to
elaborate two relatively autonomous and contrasting perspectives on math-
ematical knowledge, perspectives which are useful for analyzing in more
detail the complex conditions influencing the genesis and the development
of theoretical knowledge. Within the historical analysis as well as within the
interactive episode special problems arise; but in both domains a similar
difficult interplay between epistemological and social aspects comes up
when investigating the unfolding of knowledge meaning. This relation
receives different emphasis in the two domains, and therefore a contrasting
comparison contributes to a mutual sharpening of the analysis.

2. THE RECIPROCAL RELATIONSHIP BETWEEN PROBABILITY
AND CHANCE

Do the formal definitions contain the complete meaning of mathematical
knowledge and mathematical concepts? Can an elementary mathematical
theory be deduced precisely and in all detail from defined fundamental
concepts?

Elementary probability theory is an appropriate example with which to
analyze fundamental epistemological problems of mathematical knowledge.
In the early history of probability, games of chance provided simple and
ideal situations in which both a direct form of chance and a concrete
structurization of regular aspects were manifested in the physical symmetry
of the chance devices and their use (cf. Hacking, 1975; Maistrov, 1974).
Tossing the die is an original form of chance and disorder for which
possibilities of regular modelling were nevertheless offered by the physical
symmetry (assumed to be ideal). While neglecting mathematically precise
definitions, games of chance constituted a "concrete" elementary concept
of probability as a prediction with regard to the occurrence or non-
ocurrence of certain events, and to determining gradual certainty.

The fundamental relationship between chance and regularity, between
irregular, random phenomena on the one hand, and the mathematical ideas