SUBJECT MATTER KNOWLEDGE FOR TEACHING AND THE CASE OF FUNCTIONS

ABSTRACT. Interest in teachers' subject matter knowledge has arisen in recent years. But most of the analysis has been general and not topic-specific. This paper shows how one may approach the question of teachers' knowledge about mathematical topics. It demonstrates the building of an analytic framework of subject matter knowledge for teaching a specific topic in mathematics and then uses the concept of function to provide an illustrative case of a paradigm for analyzing subject matter knowledge for teaching. The choice of the aspects, which form the main facets of the framework, was based on integrated knowledge from several bodies of work: the role and importance of the topic in the discipline of mathematics and in the mathematics curriculum; research and theoretical work on learning, knowledge and understanding of mathematical concepts in general and the specific topic in particular; and research and theoretical work on teachers' subject matter knowledge and its role in teaching. An application of the framework in the case of the concept of function is described and illustrated by anecdotes drawn from a study of prospective secondary teachers' knowledge and understanding of functions.

INTRODUCTION

Mathematics educators today are concerned with the way mathematics is taught. They call for making a change in the way teachers teach to emphasize teaching for understanding and meaningful learning (e.g., Davis, 1986; Educational Technology Center, 1988; Lampert, 1988; Lappan and Schram, 1989; NCTM, 1989a; Peterson, 1988; Resnick, 1987; Romberg, 1983; Schoenfeld, 1987). The teacher's role is to help the learner achieve understanding of the subject matter. But in order to do so the teachers themselves need to have solid knowledge of the subject matter. A teacher who has solid mathematical knowledge for teaching is more capable of helping his/her students achieve a meaningful understanding of the subject matter. Subject matter knowledge is only one component of the knowledge of a well prepared teacher - nevertheless - an important one.

Recent reform efforts (e.g., Carnegie Task force, 1986; Holmes Group, 1986; NCTM, 1989b) are designed to improve professional teacher education. One of the goals of the current attempts to reform teaching is to strengthen the subject matter preparation of teachers: "...[prospective teachers] will be expected to pass an examination demonstrating their mastery of the subject they will teach" (Holmes Group, 1986). At the same time, interest in defining and analyzing what subject matter knowledge for teaching means has arisen.

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Conceptions of teachers' subject matter knowledge have changed throughout the years. At the beginning of this century, Dewey (1904) described teacher subject matter knowledge in qualitative terms which did not provide a straightforward way of measuring or evaluating knowledge. When process-product research on teaching became popular, teacher subject matter knowledge was defined in quantitative terms – by the number of courses taken in college or teachers' scores on standardized tests (Ball, in press; Wilson, Shulman, and Richert, 1987). But these "measures" are problematic, since they do not represent teachers' knowledge of the subject matter. Shulman's (1986) Presidential Address at the 1985 annual meeting of the American Educational Research Association in Chicago, signaled a return to a definition of teachers' subject matter knowledge in qualitative terms. Other scholars today also write about teachers' subject matter knowledge in qualitative terms (Ball, in press; Leinhardt and Smith, 1985; Tamir, 1987; Wilson et al., 1987). Defining teachers' subject matter knowledge not by the number of courses they have taken or their success on standardized tests, but by analyzing what it means to know mathematics, has some promise to contribute to the improvement of the quality of subject matter preparation of teachers and therefore the quality of teaching and learning.

Still, analyzing what teachers' subject matter knowledge means in general in mathematics, does not inform us of what subject matter knowledge teachers need to have in order to teach a specific piece of mathematics. While qualitative analysis of teachers' subject matter knowledge has brought us one step forward from a simplistic list of competencies that served as a criterion to knowledge, such analyses miss specific characteristics of knowledge needed for teaching a specific mathematical topic.

This paper shows how one may approach the question of teachers' knowledge about mathematical topics. It demonstrates the building of an analytic framework of subject matter knowledge for teaching a specific topic in mathematics. First, the development of a general framework is discussed, emphasizing the general and guiding principles of the analysis. Then, an application of the framework in the case of the concept of function is described to provide an illustrative case of a paradigm for analyzing subject matter knowledge for teaching. Anecdotes drawn from a cross-institutional study of prospective secondary teachers' knowledge of functions (Even, 1989) are used as illustration for clarifying the framework and pointing out weaknesses in existing teacher knowledge.