Comparison of Vander Lugt and Joint Transform Correlators

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Abstract. A generalized optical correlator is used to analyze the performances of the Vander Lugt and the joint transform architectures. Noise performance and the effects caused by the differences between the input and the reference functions are analyzed. It is found that the modulation index of the joint transform filter decreases due to the presence of noise as well as the differences between the input and the reference functions. However, it is found that, in general, the joint transform correlator would perform better than the Vander Lugt correlator and certainly has the advantage for real-time implementation.

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Two commonly used techniques for optical pattern recognition are the Vander Lugt [1] and the joint transform [2, 3] correlators. With the Vander Lugt correlator, investigators have used a variety of techniques to synthesize multiplexing matched filters, such as the frequency [4], the angular [5], the color multiplexing [6], the holographic lens array [7] and the rotating grating technique [8]. However, the joint transform architecture combined with a TV camera such as vidicon and a CCD can be operated in a real-time mode with ease [9, 10]. Nevertheless, the application of spatial light modulators to both Vander Lugt and joint transform correlators provide significant improvement of the system operation [8–13]. We shall, in this paper, analyze the basic performance of the Vander Lugt (VLC) and the joint transform (JTC) correlators. In the following, we first present a generalized optical correlator for both architectures. The performance and limitations of these correlators are evaluated. Multiplicative noise introduced by the spatial light modulators has been used for the analyses. The effects of the noise perturbation and the differences between the input and the reference functions on modulation index of the correlation filter are calculated. It is shown that the JTC can generally perform better than the VLC, particularly for real-time implementation.

1. A Generalized Optical Correlator

A generalized optical correlator can be described by dividing the system performance into two parts; namely, the correlation filter construction and the correlation operation. Notice that the term, "correlation filter," used here is for both the matched filter (MF) and the joint transform hologram (JTH). The construction of these filters is basically using the same procedure as shown in Fig. 1, where an input transparency \( h(x, y - b) \) and a reference function \( g(x, y + b) \) at the input plane \( P_1 \) are illuminated by a collimated coherent light beam with wavelength \( \lambda \). The intensity distribution of the joint Fourier spectrum at \( P_2 \) is given by

\[
I(u, v) = |H|^2 + |G|^2 + HG^* e^{-2\pi i ub} + H^* G e^{2\pi i vb},
\]

(1)
where $2b$ is the mean separation of the two functions and $u$ and $v$ represent the spatial frequency coordinates. If a photographic plate is inserted in plane $P_2$, the amplitude transmittance of the recorded transparency is given by

$$t = k|H|^2 + |G|^2 + kH^*G e^{-i4\pi vb} ,$$

(2)

where $k$ is a proportionality constant. We note that if a pinhole replaces the input object $h$, then (2) can be written as

$$t_m = k(A^2 + |G|^2) + kAG^* e^{-i4\pi vb} + kAG e^{i4\pi vb} ,$$

(3)

which is the well-known Vander Lugt matched filter. It is apparent that the MF can be generated by a joint transform architecture.

The principle of the correlation operation can be seen by placing a MF in $P_2$ of an optical processing system shown in Fig. 2. This arrangement is known as a Vander Lugt correlation.

Assuming that the input object is $f(x,y)$, then the complex light distribution behind the correlation filter is given by

$$U_2(u,v) = kF(|H|^2 + |G|^2) + kFH^*G e^{-i4\pi vb} + kFH e^{i4\pi vb} .$$

(4)

The corresponding output light field at $P_3$ is

$$U_3(x,y) = U_0 + kf(x_3,y_3)g^*(-x_3,-y_3)\delta(x_3,y_3-2b) + kf(x_3,y_3)g(x_3,y_3)\delta(x_3,y_3+2b) ,$$

(5)

where $\delta$ denotes the convolution operation, $U_0$ represents the amplitude distribution centered around the optical axis and the second and third terms are diffracted around $(0,2b)$ and $(0,-2b)$ respectively. We note that in the filter construction step, we replace $h(x,y)$ by a delta function, i.e., $h(x,y-b) = \delta(x,y-b)$, the second term of (5) becomes

$$R(x_3,y_3-2b) = \int \int f(\xi,\zeta)g^*(\xi-x_3,\zeta-y_3+2b)d\xi d\zeta ,$$

(6)

which is the cross correlation function between $f$ and $g$. It is apparent that the cross-correlation function $R$ would be diffracted around $(0,2b)$.

Similarly, for the third term of (5), it becomes

$$R(x_3,y_3+2b) = \int \int f(\xi,\zeta)g(x_3-x_3,\zeta-y_3+2b)d\xi d\zeta ,$$

(7)

which is diffracted around $(0,-2b)$.

On the other hand, if the object function $f(x,y)$ in the signal processing step is a $\delta$ function, the second and the third terms become

$$R(x_3,y_3-2b) = h(x_3,y_3)g^*(-x_3,-y_3)\delta(x_3,y_3-2b) ,$$

(8)

$$R(x_3,y_3+2b) = h^*(-x_3,-y_3)g(x_3,y_3)\delta(x_3,y_3+2b) ,$$

for which two cross-correlation functions of $h$ and $g$ are diffracted around $(0,2b)$ and $(0,-2b)$, respectively.

Since the input object is a delta function, as shown in Fig. 2, the correlation filter is illuminated by a collimated light beam. We see that no alignment is required for the correlation filter, and the process is known as a joint transform correlation.

2. Noise Models

Needless to say, the correlation detection would be severely degraded by noise. Noise in a coherent optical correlator generally exists in the input transparency, in the spatial filter, and in the system. We shall consider here, however, the noise in the input plane and in the spatial filter. Since spatial light modulators (SLM) are used as the input transducer and the correlation filter for real-time operation, the noise generated in these devices should be analyzed. For simplicity, the phase noise and amplitude noise will be treated separately in our analyses. Moreover, the phase noise generally comes from the surface variation of the SLM and the amplifier noise is due to the nonuniform transmittance of the device. Thus the noise at the input of an optical correlator may be written as

$$N(x,y) = |N(x,y)|e^{i2\pi p(x,y)} ,$$

(9)

where $|N(x,y)|$ and $p(x,y)$ are amplitude and phase noise, respectively.

By referring to the generalized correlator of Fig. 1, we assume that two magneto-optic devices (MODs) are used as input transducers and a liquid crystal light valve (LCLV) is employed as a square law converter in the Fourier plane [9, 10]. Since the MOD is a binary transmission device in which the data written into the