Epicycles, Eccentrics, and Ellipses: The Predictive Capabilities of Copernican Planetary Models

C. A. GEARHART

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Abstract

A theoretical analysis of the potential accuracy of early modern planetary models employing compound circles suggests that fairly simple extensions of those models can be sufficiently accurate to meet the demands of Tycho Brahe’s observations in both ecliptic longitude and latitude. Some of these extensions, such as the substitution of the true sun for the mean sun, had already been taken by Kepler before he abandoned circular models. Other extensions, involving one or two extra epicycles, were well within the mathematical capabilities of sixteenth-century and seventeenth-century astronomers. Hence neither the failure of astronomers before Kepler to correct errors in planetary positions nor Kepler’s decision to abandon circular models was a consequence of inherent limitations in those models.

The hold of the circle as a predictive planetary model on even so late a figure as Kepler was strong indeed. Kepler’s initial attempts to describe the orbit of Mars, the planet that because of its large eccentricity was most troublesome to pre-Keplerian astronomy, involved the use of Ptolemy’s equant, in which the uniform motion of the planet on a circle was measured about an off-center equant point (see Figure 1a). Only after the failure of various circular equant models to predict both the ecliptic longitude and latitude of Mars within the accuracy of Tycho Brahe’s observations did Kepler reluctantly abandon the circle and adopt the path that finally led him to the ellipse. In this search Kepler insisted that planetary models should not only predict accurately but also be (to him) physically plausible. He disliked and for the most part did not use epicyclic models, because he could see no plausible physical basis for motion on an epicycle. 1

1 Kepler, Astronomia Nova (1609), cap. 4. (All references to this work were taken from Max Casper’s German translation [Munich-Berlin: R. Oldenbourg, 1929].) See also Alexandre Koyré, The Astronomical Revolution (Ithaca: Cornell Univ. Press, 1973), p. 177.
Fig. 1. Copernican and equant models. The planet is located at P, and angle ASP is the heliocentric longitude in both. Figure 1a is the equant model. The sun S and the equant point Q are both removed a distance e from the center C of the deferent. The angle AQP is the mean anomaly \( \alpha \). Figure 1b is the Copernican model. The sun S is removed \( SC = 3/2e \) from the center of the deferent. Angle ACT is the mean anomaly \( \alpha \) that advances uniformly with time. The epicycle has a radius \( TP = \frac{1}{2}e \) on which the planet P rotates in a CCW sense, at a rate such that angle \( A_1TP = 2\alpha \).

And he placed great importance on a magnetic analogy, in which he imagined the planets to be maintained in their orbits by the magnetic influence of a rotating sun.\(^2\)

Neither assumption was widely shared, or widely adopted. Most of KEPLER’S immediate predecessors, from COPERNICUS onward, used epicycles freely but found the equant physically implausible, since it violated the principle of uniform circular motion. Nor did KEPLER’S magnetic analogy arouse any great enthusiasm among his contemporaries. Even the enormous improvement in predictive power afforded by the ellipse did not lead to the immediate widespread adoption of KEPLER’S new astronomy; many astronomers preferred to regard the ellipse as no more than an empirically accurate description of the orbit. The French astronomer BOULLIAU, for example, constructed elliptical orbits using circular devices, thus preserving uniform circular motion.\(^3\) NEWTON, too, who established the theoretical basis of KEPLER’S laws, remarked of KEPLER that he “knew the Orb ... to be oval and guest it to be Elliptical.”\(^4\)

Given this pre-NEWTONian reluctance to regard the elliptical path as more than an empirical description, one may ask how inevitable was the discovery of even this much by the early seventeenth century. Was it necessary to turn to

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\(^2\) KEPLER’S development of this argument is scattered throughout the *Astronomia Nova*; for a good discussion, see KOYRÉ, *Astronomical Revolution* (note 1), pp. 197–214.
