Calculation of a three-dimensional turbulent cascade flow

J. Y. Yoo, J. W. Yun

Abstract A three-dimensional steady incompressible viscous flow through a plane cascade of turbine blades has been analyzed through a numerical method based on the Navier-Stokes equation. Particular attention is paid to the prediction of secondary flows occurring due to the endwall boundary layer and the blade geometry. A standard k-ε model is used for the modelling of Reynolds stress and boundary-fitted coordinates are adopted to represent the complex blade geometry accurately. Two differencing schemes are applied to the convective terms to investigate the effect of numerical diffusion. Experimental data obtained for the flows through the Langston cascade are selected for code validation. Computed results for the velocity vectors and static pressure distributions are in good agreement with previous measurements and provide validity of this numerical method. Three-dimensional viscous flow phenomena and the distribution of total pressure loss caused by secondary flows are also reasonably well predicted.

List of symbols

- area of the control volume surface
- coefficient of the discretized equation
- product of the transformation matrix
- transformation matrix
- axial chord of the airfoil
- pressure coefficient = \((p - p_i)/\rho_i q_i^2\)
- total pressure loss coefficient = \((p_i - p)/\rho_i q_i^2\)
- mass-averaged total pressure loss coefficient
- coefficients of the standard k-ε model
- constant used in the law of the wall
- unit vectors in the Cartesian coordinate system
- mass flux through the control volume surface
- linear interpolation constant
- production term in the turbulent kinetic energy, or the contravariant velocity
- Jacobian
- turbulent kinetic energy
- characteristic length
- nondimensionalized vertical distance from the wall
- pressure
- total pressure

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Introduction

To predict and improve the performance of gas turbines, it is important to understand secondary flows and loss distribution by analyzing the flow through turbine blade passages from the standpoint of the design of blade geometry. Since the secondary flows are three-dimensional flow phenomena, it is highly desirable to be able to describe the complex three-dimensional viscous flows. Particularly, the total pressure loss within the cascade passage associated with the passage vortex has been an important problem. The passage vortex is synthesized by the horseshoe vortex formed from the inlet boundary layer, the crossflow from the endwall boundary layer within the cascade passage and the entrained fluid from the mainstream flow in the cascade passage (Langston 1980). Salvage (1974) and Sieverding (1985) reviewed previous studies on secondary flows and the corresponding losses. Langston et al. (1977) performed an extensive and detailed experimental study, including elaborate flow visualizations, on the complex three-dimensional flow within a large scale plane cascade of turbine airfoils. They showed both quantitatively and qualitatively the characteristics of secondary flow phenomena. Later, Langston (1980) performed a measurement of the endwall boundary layer and showed the characteristics of the three-dimensional boundary layer. Moore and Ransmayr (1984) conducted an experimental investigation to study the effect of the shape of the leading-edge on overall losses in a plane cascade of turbine blades. Moore and Smith (1984) used an ethylene detection technique to confirm the existence of the horseshoe vortex and to study the total pressure loss in the downstream region. Moore and Adhye (1985) used five-hole and three-hole probes to investigate experimentally the loss mechanism and the decay of the passage vortex. They further found out that more than one-third of the losses occurred downstream of the trailing edge.

Since the three-dimensional turbulent flow through a turbine blade passage is a very complicated phenomena with streamline curvature, rotation, severe pressure gradient, the anisotropy of turbulence and so forth, it is important to provide appropriate closure models for adequate numerical prediction of such a complex flow. Along with increasing computer capabilities, some developments in three-dimensional viscous flow calculations for turbomachinery have also been made. For example, Hah (1984) analyzed the three-dimensional turbulent flow inside turbine blade rows using an algebraic Reynolds stress model modified for the effects of the streamline curvature and the rotation. Moore and Moore (1985) used the Prandtl mixing-length turbulence model for three-dimensional turbulent flows

\( q \) magnitude of the velocity vector
\( Re \) Reynolds number
\( Res_\phi \) residual of the discretized equation
\( S_\phi \) production term of the governing equation for \( \phi \)
\( T_w \) wall shear stress
\( U, V, W \) mean velocity components in the \( x, y \) and \( z \) directions
\( u, v, w \) fluctuation velocity components in the \( x, y \) and \( z \) directions
\( X, Y, Z \) nondimensionalized Cartesian coordinate system
\( x, y, z \) Cartesian coordinate system
\( \Gamma_\phi \) diffusion coefficient of \( \phi \)
\( \delta \) Kronecker delta
\( \delta p \) small pressure difference
\( \delta x \) width of the control volume
\( \epsilon \) turbulent kinetic energy dissipation rate
\( \kappa \) von Karman constant
\( \mu \) viscosity
\( \mu_{eff} \) effective viscosity
\( \mu_t \) eddy viscosity
\( \xi, \eta, \zeta \) curvilinear coordinate system
\( \rho \) density
\( \sigma_{\phi}, \sigma_k \) constants for the standard \( k-\epsilon \) model
\( \phi \) general flow variable

Subscripts
\( E, W, P, \text{ etc.} \) neighboring grid points
\( e, w, p, \text{ etc.} \) control volume surfaces
\( i, j \) components of a vector or a tensor
\( * \) assumed values
\( ' \) corrected values
\( 1 \) values at the inlet surface
\( 1, 2, 3 \) components of a vector

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