There are two natural ways in which the modal concept of necessity can be treated in formal languages. One way, which is advocated by Quine, is to treat necessity as a semantical predicate which is attributed to statements considered as syntactical entities. Under this development, the notational forms of sentences are designated by singular terms in the language, and the necessity predicate attaches to these terms. The second way is to treat necessity as a logical operator which attaches to statements directly, in the same manner as the logical operator of negation. Quine labels these the “first” and “second” grades of modal involvement, respectively, and considers them to be fundamentally distinct. Only the first grade is deemed philosophically legitimate, and the latter grade is to be condoned only insofar as it can be reduced to the former.

But Montague has demonstrated, via an extension of Tarski’s theorem, that treatment of necessity as a predicate of sentences will lead to inconsistency, if the language in which this is accomplished is capable of unrestricted self-reference. In such a language, the diagonal lemma will yield a sentence $G$ which is true if and only if it is not necessary. And this sentence, in conjunction with two basic properties which seem fundamental to our notion of necessity (namely that if $G$ is necessary, then it is true, and that if $G$ is provable, then it is necessary), will lead to the provable contradiction that $G$ is both necessary and not necessary.

Montague takes the philosophical import of this demonstration to be that necessity cannot be consistently treated as a metalinguistic predicate of sentences, without giving up at least one of the aforementioned properties; but each seems essential to our intuitive concept of necessity. Thus he concludes that our intuitive concept of necessity cannot be formally captured as a predicate, and that Quine’s program of reduction

*Philosophical Studies* 52 (1987) 33—47.
Montague neglects the alternate possibility, which is to treat necessity as a metalinguistic predicate in a language which is not capable of unrestricted self-reference. In ‘An Immaculate Conception of Modality’, Brian Skyrms develops this alternate possibility. He constructs an infinite hierarchy of meta-languages in which, at each level, necessity is treated as a predicate which attaches to names of sentences at the preceeding levels. This hierarchy is consistent, and possesses a metalinguistic counterpart for every modal sentence of a generic language in which necessity is treated as an operator. And if the necessity predicate is interpreted as attributing the semantical property of validity, then exactly those formulas which are metalinguistic counterparts of the theorems of S5 turn out true in all models of the hierarchy. He thereby shows that Quine’s second grade of modal involvement can be formally translated in terms of the first, and hence that Montague’s results simply block one particular approach to construing necessity as a predicate.

In response to Skyrms’ construction, Richard Otte has argued that the hierarchical approach avoids inconsistency only because of an artificial limitation imposed on the naming function. Skyrms does not use Gödel numbers as names of sentences, but rather utilizes a quotation function Q. And the formation rules are such that this function symbol is always syntactically joined with the necessity predicate *, so that expressions in which Q appears by itself are not well formed. Otte maintains that this seemingly ad hoc restriction is the crucial factor in averting paradox, and that if terms formed by the naming function were allowed to appear without being prefixed by the modal predicate (as is the case with the Gödel numbers used in Montague’s demonstration), then inconsistency would immediately ensue. And the purportedly unavoidable presence of this artificial restriction is taken as evidence in favor of the philosophical position that necessity should be treated as an operator.

In the present paper, several modified versions of Skyrms’ hierarchy will be constructed, in which the restriction noted by Otte will not obtain. Names formed by the quotation function will be treated as ordinary singular terms, which can therefore occur in isolation from the modal predicate. The consistency of these languages will establish that the syntactic fusion of * and Q is merely an incidental feature of