More on Insurance as a Giffen Good

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Abstract

In this article, we generalize the Hoy and Robson (1981) analysis and provide a necessary and sufficient condition for insurance not to be a Giffen good. The condition gives a bound for the variation of absolute risk aversion that permits the wealth effect to be always dominated by the substitution effect.

In a well-known article, Mossin (1968) noted that insurance is an inferior good under decreasing absolute risk aversion. This contribution has generated some further investigations on the nature of the insurance activity (see, for example, Marshall, 1974; Hoy and Robson, 1981; Dionne and Eeckhoudt, 1984; Borch, 1986; Doherty and Dionne, 1989.) More particularly, Hoy and Robson (1981) derived an explicit theoretical condition under which insurance is a Giffen good for the class of constant relative risk-aversion functions. They showed that a necessary condition for insurance to be a Giffen good is that the coefficient of relative risk aversion be greater than one.

In this article, we generalize their analysis and give necessary and sufficient conditions to sign variations of optimal insurance coverage for changes either in initial wealth or in the insurance rate. Our analysis extends the important comparative statics results of Cheng, Magill, and Shafer (1987) to the insurance literature.

After presenting the model in section 1, we show in section 2 how an increase in initial wealth affects the optimal level of insurance coverage. We then present, in the following section, a necessary and sufficient condition for insurance not to be a Giffen good. Under decreasing absolute risk aversion, there are two opposite effects when the insurance rate increases: a negative substitution effect and a positive wealth effect. Proposition 2 gives a bound for the variation of absolute risk
aversion that permits the wealth effect to be always dominated by the substitution effect. Proposition 3 shows how the Hoy and Robson (1981) contribution is related to our results.

1. The model

Let us consider a risk-averse individual with an initial wealth $W_0$ subject to a random loss $L$ ($0 < L < W_0$) with a density function $f(L)$. His utility function is increasing and strictly concave in wealth ($U'(W) > 0$ and $U''(W) < 0$). Market insurance is available. The individual can buy coverage $\alpha L$ ($0 < \alpha < 1$) for a premium of $\alpha P$ where $\alpha$ is the rate of insurance coverage, $\lambda$ ($\lambda > 1$) is a loading factor, $E(L)$ is the expected loss, and $P = \lambda E(L)$.

The optimal level of insurance coverage solves the following problem:

$$\max_{\alpha} E[U(W_0 - L + \alpha(L - \lambda E(L)))] = 0. \quad (1)$$

The first-order condition for an optimal $\alpha$ (denoted $\alpha^*$) is

$$E[U'(W_0 - L + \alpha(L - \lambda E(L)))(L - \lambda E(L))] = 0. \quad (2)$$

Equation (2) implies that $0 < \alpha^* < 1$ for $\bar{P} > P > E(L)$ where $\bar{P} = \bar{\lambda} E(L)$ solves

$$E[U(W_0 - L + \alpha^*(L - \bar{\lambda} E(L))] = E[U(W_0 - L)] = 0. \quad (3)$$

In the remainder of the article, we assume that $1 < \lambda < \bar{\lambda}$ so that $1 > \alpha^* > 0$. (See Borch, 1986, for the analysis of negative amounts of insurance.) Given concavity of $U(\cdot)$, equation (2) is also sufficient for a maximum.

For the sake of notational convenience, define $W = W_0 - L + \alpha(L - \lambda E(L))$, and notice that equation (2) is equivalent to

$$(1/\alpha^*)E[U'(W)(W + L - W_0)] = 0. \quad (4)$$

2. Changes in initial wealth

Mossin (1968) showed that if an insured has decreasing risk aversion, then $\alpha^*$ decreases when wealth increases. In this section, we present necessary and sufficient conditions to sign the variation of $\alpha^*$ when wealth changes, first at a given initial wealth level and then for all initial wealth levels. We will use the term $A(W) = -U''(W)/U'(W)$ to denote the Arrow-Pratt index of absolute risk aversion.