Optimal Filtering of Nerve Signals*

M. N. Oğuztöreli and R. B. Stein
Departments of Mathematics and Physiology, University of Alberta, Edmonton, Canada

Abstract. Recording from multiple electrodes at different sites along a peripheral nerve permits the application of powerful filtering methods to extract the activity of populations of fibres within the nerve which differ in temporal or spectral characteristics. The design of optimal linear filters is initially treated as a general problem in the calculus of variations in which the signals from one population of nerve fibres are extracted so as to minimize those from a second population of nerve fibres or from other sources (noise). A particularly important application arises when the signals at two electrodes are related by weighting functions. In the simplest example the weighting function represents the time delay for nerve impulses to conduct from one electrode to the other, but explicit results are also derivable when there are a range of conduction delays with probabilities distributed according to well-known functions such as the sinc² function.

I. Introduction

Mammalian nerves typically contain hundreds or thousands of nerve fibres, each transmitting up to a thousand nerve impulses a second. Considerable progress has been made in sampling the activity of one, or in some cases several nerve fibres simultaneously in behaving animals (e.g., Evarts, 1966; Mountcastle et al., 1975). With careful experimental studies over time the activity of a substantial population of single units can be recorded. Nonetheless, this population represents only a small and sometimes biased sample of the total population within the nerve.

On the other hand, recording the activity of a whole nerve gives the sum of impulses signalling various, sometimes contradictory events, and it has been difficult to obtain meaningful results from this gross activity except under special conditions [e.g., evoked potential studies with light flashes to the visual system, or clicks to the auditory system, Kiloh et al. (1972) and Picton et al. (1974)]. However, recording from more than one point along a nerve offers some hope, since motor and sensory signals will be conducting in opposite directions and even within the population, for example, of sensory signals various classification schemes have long been developed relating the size and conduction velocity of nerve fibres to their function (Ganong, 1975).

In addition methods have recently been developed for recording chronically from mammalian nerves (DeLuca and Gilmore, 1975; Stein et al., 1975; Hoffer and Marks, 1976), and these methods can be applied to simultaneous recordings from several nerves or several points along one nerve. In the simple example shown in Figure 1, if we record from two electrodes, one near the peripheral end of a nerve and one near the central end, the order of occurrence of impulses from two nerve fibres may be different. The sensory impulse in Figure 1 will occur earlier at the peripheral (u₁₁) than at the central electrode (u₂₁) while the motor impulse will occur sooner at the central electrode, because of its different direction of conduction. In this diagram, as later in the paper, the first index will refer to the electrode whereas the second index will refer to the recorded impulse of a unit potential. Because of the time delays involved, approaches involving cross-correlation have been used to distinguish sensory and motor activity even when many nerve fibres are active and the single unit potentials superimpose on each other (Heetderks and Williams, 1975).

In general the amplitude and waveform of the nerve impulses may be different at the two electrodes and a
more general approach is suggested based on the theory of optimal linear filtering. Roberts and Hartline (1975) have shown that this theory could be applied to distinguish potentials from up to five single nerve fibres in a small invertebrate nerve. In the present paper we will provide a fuller mathematical treatment of this problem. The basic formulation of the problem mathematically is carried out in the next section and its solution as a variational problem in the third section. We have extended the treatment to consider not only single nerve fibres but rather whole populations of nerve fibres. Several special cases are considered in which the populations are distributed according to some simple mathematical functions.

A schematic representation of the problem is shown in Figure 2. To avoid confusion with the Dirac δ-function or impulse function in the mathematical derivation, we will refer to \( u_{1,1} \), \( u_{1,2} \), etc. as unit potentials or simply units, rather than as impulses from single nerve fibres or populations of nerve fibres. Electrode 1 will record signals from various sources given by the sum \( x_1(t) \):

\[
x_1(t) = u_{1,1}(t) + u_{1,2}(t) + v_1(t),
\]

whereas, as before, \( u_{1,1}(t) \) may refer to sensory units and \( u_{1,2}(t) \) may refer to motor units, and \( v_1(t) \) refers to other extraneous signals or noise picked up by the first electrode.

The problem then is to design filters \( f_{1,1}(t) \) and \( f_{2,1}(t) \) which optimally extract the signals of the first unit's potentials at the two electrodes and a corresponding set \( f_{1,2}(t) \) and \( f_{2,2}(t) \) to optimally extract the signals of the second unit's potentials from that of the first unit and from the noise. Then, the output signal for the first unit will be

\[
y_1(t) = f_{1,1}(t) * x_1(t) + f_{2,1}(t) * x_2(t),
\]

where the * represents a convolution operation which is defined in the usual way (see Section II). Similarly, the output signal for the second unit will be

\[
y_2(t) = f_{1,2}(t) * x_1(t) + f_{2,2}(t) * x_2(t).
\]

Although the formulation of the filtering process will be carried out here for two channels, one could easily extend it any number \( m \) electrodes and \( n \) units. Roberts and Hartline (1975) suggest from their experimental studies that the filtering is most effective if \( n \) does not exceed \( m \).

II. Description of the Filtering Process

We begin the mathematical description of the 2-channel linear filtering process outlined in the previous section by listing the symbols used throughout this work. For the further terminology and symbols we refer to Bracewell (1965), Kanasewich (1973), Robinson (1962, 1967), and Treitel (1970).

\( t \) = time (s)

\( j \) = index for electrodes \( (j = 1, 2) \)

\( k \) = index for units \( (k = 1, 2) \)

\( a^* \) = complex conjugate of \( a \)

\( \text{Re } a \) = real part of \( a \)

\( \text{Im } a \) = imaginary part of \( a \)

\( a(t) * b(t) \) = convolution of \( a(t) \) with \( b(t) \)

\( a(t) \otimes b(t) \) = cross-correlation (pentagram) of \( a(t) \) and \( b(t) \)

\( A(s) \) = Fourier transform of \( a(t) \)

\( a(t) = \text{inverse Fourier transform of } A(s) \)

\[
\int_{-\infty}^{\infty} a(t) e^{-2\pi i s t} dt
\]

\[
\int_{-\infty}^{\infty} A(s) e^{2\pi i s u} ds
\]