Abstract We have analysed the development of shear bands during the perforation of a steel plate by a massive rigid punch. The contact surfaces are modeled as smooth and the steel is modeled as a thermoviscoplastic material that hardens with an increase in the plastic strain and plastic strain-rate but softens due to the rise in its temperature. The effect of punch speed, the clearance between the punch and the back supports, and the radius of the periphery of the punch nose on the development of bands is delineated.

1 Introduction
Even though Tresca (1878) reported the development of shear bands during the hot forging of platinum more than a century ago, the activity in this field picked up since the time Zener and Hollomon (1944) observed 32 μm wide shear bands during the punching of a hole in a low carbon steel plate. Zener and Hollomon also pointed out that during the punching process, heat produced because of the intense plastic deformations of the material soften it and once this softening equals the hardening of the material due to strain and strain-rate effects, it becomes unstable. The study of shear bands is important because once these bands have developed, subsequent deformations of the body are concentrated in these narrow regions and the strength of the rest of the body is not fully utilized. Also, shear bands precede shear fractures and are the primary mode of failure in ductile materials under dynamic loading. Subsequently Moss (1981) conducted tests similar to those of Zener and Hollomon (1944) and reported that strain-rates of the order of 10^5/sec occur within the shear band. Chou et al. (1991) have performed controlled penetration tests and have measured the length of the band ahead of the punch surface.

2 Formulation of the problem
We use a cylindrical coordinate system to analyse axisymmetric deformations of a steel plate impacted at normal incidence by a massive rigid cylindrical rod and assume that the contact surfaces are smooth; a schematic sketch of the problem studied is shown in Fig. 1 wherein dimensions of different parts are also given. We use an updated Lagrangian description of motion. Equations governing thermomechanical deformations of the steel plate are

\[ (p\mathbf{\dot{J}}) = 0, \]  
\[ \rho \mathbf{\dot{\mathbf{v}}} = \text{Div} \mathbf{T}, \]  
\[ \rho \dot{\mathbf{e}} = -\text{Div} \mathbf{Q} + tr(T\mathbf{F}^T). \]

Equations (1), (2) and (3) express, respectively, the balance of mass, balance of linear momentum and the balance of internal energy. In them \( \rho \) is the present mass density of a material particle whose mass density in the reference configuration is \( \rho_0 \). \( J = \det \mathbf{F} \) is the deformation gradient, \( \mathbf{x}(\mathbf{X}, t) \) gives the position of the material particle \( \mathbf{X} \) at time \( t \), \( \mathbf{v} = \mathbf{\dot{x}} \) gives the velocity of the material particle \( \mathbf{x} \), a superimposed dot indicates the material time derivative, \( \mathbf{T} \) is the first...
Piola-Kirchhoff stress tensor, \( e \) is the specific internal energy, and \( Q \) the heat flux measured per unit reference area. The operator \( \text{Div} \) indicates the divergence operator with respect to coordinates in the reference configuration. Equations (1)–(3) are supplemented with the following constitutive relations.

\[
\sigma = -p(\rho) 1 + 2\mu \tilde{D}, \quad p(\rho) = K(\rho/\rho_0 - 1) \tag{4}
\]

\[
2\mu = \frac{\sigma_0}{\sqrt{3 I}} (1 + b I)\mu (1 - \nu \theta) \left( 1 + \frac{\psi}{\psi_0} \right)^n, \quad 2I^2 = \text{tr}(\tilde{D} \tilde{D}^T), \tag{5}
\]

\[
\dot{\psi} = 4\mu I^2 \left( 1 + \frac{\psi}{\psi_0} \right) \sigma_0, \quad \tilde{D} = D - \frac{1}{4} \text{tr}(D) I, \tag{6}
\]

\[
T = J \sigma (F^{-1})^T, \quad Q = J q (F^{-1})^T, \quad q = -k \text{grad} \theta, \tag{7}
\]

\[
\rho_0 \dot{\rho} = \rho_0 c \dot{\theta} + p \rho \dot{\rho} + \rho. \tag{8}
\]

Here \( \sigma \) is the Cauchy stress tensor, \( K \) the bulk modulus, \( \sigma_0 \) the yield stress in a quasistatic simple tension or compression test, \( \psi \) an internal variable that describes the hardening of the material, \( k \) the thermal conductivity, \( c \) the specific heat and \( \theta \) the rise in the temperature of a material particle. From (4), and (5), it follows that

\[
\left( \frac{3}{2} \text{tr}(s s^T) \right)^{1/2} = (1 + b I)\mu (1 - \nu \theta) \left( 1 + \frac{\psi}{\psi_0} \right)^n. \tag{9}
\]

That is the material obeys the von Mises yield criterion and the yield stress depends upon the strain-rate, temperature and the work-hardening parameter \( \psi \). The constitutive relation (4) with \( \mu \) given by (5), was first proposed by Batra (1988). In it, \( \sigma_0 \), \( b \), \( m \), \( \psi_0 \) and \( n \) are material parameters; \( b \) and \( m \) describe the strain-rate hardening of the material, \( \nu \) its thermal softening and \( \psi_0 \) and \( n \) its work-hardening. Once \( (1 - \nu \theta) \) equals zero, the material behaves like an ideal fluid. We note that in our work the volumetric deformations are considered to be elastic, the distortional or shear deformations are taken to be plastic, there is no unloading considered, and a material point is assumed to deform plastically at all times; however, the plastic strain-rate is extremely small for low values of the effective deviatoric stress. Also, there is no failure or fracture criterion considered.

The plate is taken to be initially stress free, at rest, at a uniform temperature and the initial work-hardening is set equal to zero. The rigid penetrator or the punch is assumed to be moving at a uniform speed \( V_0 \) and is taken to be huge so that its speed can be assumed to be uniform during the perforation process.

Here we assume that all of the plastic working is converted into heating or equivalently have taken the Taylor-Quinney parameter equal to \( 1 \). Batra and Adulla (1994) have shown that a lower value of the Taylor-Quinney parameter delays the initiation of a shear band but has no effect on the qualitative nature of results.

For the boundary conditions we take \( v = 0 \) and \( q \cdot n = 0 \) at the plate particles abutting the rigid supports, and \( (v \cdot n) n = V_0 n, q \cdot n = 0 \) at the smooth target/penetrator interface where \( n \) is a unit normal at a point on the interface. Thus the plate is assumed to be glued to the rigid stationary back supports and there is no interpenetration of the plate material into the punch. Should a gap develop between the plate and punch surfaces, the plate particles there are assumed to be traction free and thermally insulated. Because of the rather very short time duration of the punching process, the assumption of no heat transfer from the target into either the supports or the penetrator is a reasonable one.

The above-stated problem is highly nonlinear and too difficult to solve analytically; therefore, we seek its approximate solution by the finite element method.

3 Computation and discussion of results
In order to compute numerical results, we assigned following values to various parameters for the steel.

\[
\sigma_0 = 792 \, \text{MPa}, \quad K = 157 \, \text{GPa}, \quad b = 10000 \, \text{sec},
\]

\[
v = 0.66 \times 10^{-5} \, \text{C}^{-1}, \quad m = 0.01, \quad n = 0.09, \quad k = 50 \, \text{W/m} \, ^\circ\text{C},
\]

\[
\rho_0 = 7840 \, \text{Kg/m}^3, \quad \theta_0 = 25 \, ^\circ\text{C}, \quad \psi_0 = 0.017, \quad c = 477 \, \text{J/Kg} \, ^\circ\text{C}
\]