Abstract. The paper discusses "regularisation" of dualities. A given duality between (concrete) categories, e.g. a variety of algebras and a category of representation spaces, is lifted to a duality between the respective categories of semilattice representations in the category of algebras and the category of spaces. In particular, this gives duality for the regularisation of an irregular variety that has a duality. If the type of the variety includes constants, then the regularisation depends critically on the location or absence of constants within the defining identities. The role of schizophrenic objects is discussed, and a number of applications are given. Among these applications are different forms of regularisation of Priestley, Stone and Pontryagin dualities.

Key words: Priestley duality, Stone duality, Pontryagin duality, character group, program semantics, Bochvar Logic, regular identities.

1. Introduction

Priestley duality [26] [27] between bounded distributive lattices and ordered Stone spaces may be recast as a duality between unbounded distributive lattices and bounded ordered Stone spaces [6, p.170]. In joint work with Gierz [7], and using a range of ad hoc techniques, the first author combined this form of Priestley duality with the Ponka sum construction to obtain duality for distributive bisemilattices without constants. Such algebras have recently found application in the theory of program semantics [15] [28]. The variety of distributive bisemilattices without constants is the regularization of the variety of unbounded distributive lattices, i.e. the class of algebras, of the same type as unbounded lattices, satisfying each of the regular identities satisfied by all unbounded distributive lattices. In this sense, the duality of [7] may be described as the regularization of Priestley duality without constants.

For a type with constants, there are various forms of regularization according to the location or absence of the constants within identities. Part of the motivation for the current work is the desire to apply these versions of regularization to Priestley duality in its original form, i.e. as a duality for bounded distributive lattices. Indeed the resulting algebras, distributive bisemilattices with constants, are more readily suited to the applications in program semantics. Another motivation for the current work is the desire
to extend traditional Pontryagin duality by regularization. Since Pontryagin duality (between discrete abelian groups and compact Hausdorff abelian groups) involves constants, such extensions have to involve constants as well.

The ad hoc techniques of [7] did not lend themselves well to further development. As a first step, the present authors gave a general and more conceptual treatment of the regularization of a duality for algebras without constants [34]. There were two key ingredients in this treatment. The first (which was also used independently by Idziak in a different context [12]) was the interpretation of the Plonka sum as an equivalence between semilattice representations or sheaves and (total spaces of) bundles. The second was a purely categorical theorem showing how an initial duality could be enlarged to a duality between categories of representations of semilattices in the categories of the initial duality. (The initial duality was recovered from the enlarged duality as the duality for representations of trivial semilattices.) The present paper recalls the key concepts of [34], but the reader is referred there for details such as the proof of the duality theorem for semilattice representations. The proof remains valid for the improved version of this duality theorem (Theorem 4.4) formulated here with less restrictive completeness assumptions.

For algebras with constants, there are three main forms of regularization. These are known as regularization, symmetrization, and symmetric regularization. They correspond to three possible locations of a constant object in a meet semilattice viewed as a poset category: terminal, initial, or arbitrary. There are three corresponding varieties of semilattices with constant. The second section of the paper discusses duality theorems for semilattices without constants and for these three varieties of semilattices with constant. These semilattice dualities may be viewed as regularizations of (trivial) duality for trivial algebras. Semilattice duality is the basis for regularization of other dualities.

The third section starts with the analysis of identities with and without constants that leads to the various forms of regularization. It then presents the various forms of the Plonka sum construction that describe algebras in the various regularizations of a strongly irregular variety $\mathcal{V}$. For the symmetrization and symmetric regularization, the appropriate Plonka sums represent semilattices in a new category $^{2}\mathcal{V}$. This new category is itself a sort of Plonka sum of the category $\mathcal{V}$ (with homomorphisms as morphisms) and the "non-constant reduction" of $\mathcal{V}$, the variety $\mathcal{V}_+$ generated by the non-constant reducts of algebras from $\mathcal{V}$.

The fourth section begins with the concept of compatibility between dualities for $\mathcal{V}$ and $\mathcal{V}_+$. Given such compatible dualities, there is a duality for