Abstract. This paper lays out a program for analysing relevance constructively. It begins with a summary of results concerning the system $C$ of Pottinger [197a] which has entailment, relevant implication, $S4$ strict implication, and intuitionist implication among its connectives. A full working out of the motivation for $C$ will require formal analysis of informal concepts derived from the usual explanation of the meanings of the constants of intuitionist propositional logic. Formal machinery which should be adequate for the proof theoretic side of the analysis of the nonmodal part of $C$ is described in detail, and the direction in which semantical results are to be sought is indicated.

1.0. Introduction

The purpose of this paper is to describe how things stand with the project of analysing relevance constructively which was begun by the present author [4] and carried further in [7], [6], and [8]. I will begin by summarizing briefly what was done in the papers just referred to, and then I will go on to say what has been done since and what needs to be done now.

Because the paper is intended to be programmatic, proofs will not be presented and quite a number of conjectures will be put forth. No doubt some of these conjectures will turn out to be false, but it seems reasonable to hope that no falsehoods sufficient to destroy the overall scheme will turn up when the details are worked out.

I will have to assume that the reader is familiar both with the basic ideas about relevance presented by Anderson, Belnap, et al [1] and with the normalization techniques of Prawitz [9]. In addition, familiarity with [5] would be helpful, and an understanding of the lambda calculus is essential.1

2.0. The story of $C$

I will begin by laying out a formulation of the system $C$ which was first defined in Pottinger [197a]. $C$ has a primitive logical constants entailment (→), $S4$ strict implication (→), relevant implication (→),

1 For an excellent introduction to the lambda calculus and combinatory logic in general, see Hindley, Lercher, and Seldin [4].
intuitionist implication (\(\rightarrow\)), intuitionist conjunction (\(\&\)), intuitionist disjunction (\(\lor\)), and an absurdity constant (\(\mathcal{A}\)). Formulas of \(C\) are built up from these items, propositional parameters, and grouping indicators in the usual way.

If \(i\) is a positive integer, then \(^i\mathcal{A}\) is an indexed formula. \(M\) is to be either * or the empty expression. \(H, H_1, \ldots\) are to be things of one of the forms \(^i\mathcal{A}^M\) or \(^i\mathcal{A}^a\), where \(a\) is a finite set of positive integers and \(i\) is not a member of \(a\). \(P, Q, R, S, P_1, \ldots\) are to be lists of the form \(H_1, \ldots, H_n\) \((n \geq 0)\), where (1) for \(i < j \leq n\), if \(^{k_i}\mathcal{A}_i\) is the indexed formula of \(H_i\) and \(^{k_j}\mathcal{A}_j\) is the indexed formula of \(H_j\), then \(k_i\) is distinct from \(k_j\), and (2) for \(j < n\), if \(H_j\) has the form \(^{k_A}\mathcal{A}^a\) and \(m\) is a member of \(a\), then for some \(j_1 < j\), \(H_{j_1}\) has the form \(^m\mathcal{B}^M\). Sequents have the form \(P \rightarrow \mathcal{A}\) \(a\).

'\(X\)' will be used as a variable taking \(\rightarrow, \neg\), \(\rightarrow\), and \(\rightarrow\) as values. \(\rightarrow\) and \(\neg\) are strict and \(\rightarrow\) and \(\rightarrow\) are relevant. Define:

1. \(A\) is strict
2. If \(X\) is strict, then \(A \times B\) is strict.
3. If both \(A\) and \(B\) are strict, then \(A \times B\) and \(A \lor B\) are strict.

The system \(C_s\) is defined as follows. ('\(\mathcal{C}\)' is for 'combined', and '\(S\)' is for 'sequent'. A Fitch-style formulation of \(C\) is given in section 1 of [7]. Whether a Hilbert style formulation can be given is an open question.)

**Axioms**

**A1** \(P, \ ^i\mathcal{A}^M \vdash \mathcal{A}_a\)  
provided \(a \subseteq \{i\}\)

**A2** \(P, \ ^i\mathcal{A}^b \vdash \mathcal{A}_a\)  
provided \(a \subseteq b\)

**Structural Rule**

**REIT** \(P \vdash \mathcal{A}_a\) \(P, H \vdash \mathcal{A}_a\)  
provided that if \(H\) has the form \(^B\mathcal{A}^a\), then \(A\) is strict

**E Rules**

**XE** \(P \vdash A \times B_a\) \(P \vdash A_b\) \(P \vdash B_c\)  
where (1) if \(X\) is relevant, then \(c = a \cup b\), and (2) if \(X\) is not relevant, then \(c = a\)

**&E** \(P \vdash A \& B_a\) \(P \vdash A \& B_a\) \(P \vdash A_a\) \(P \vdash B_a\)