Abstract. The perfect fit of syntactic derivability and logical consequence in first-order logic is one of the most celebrated facts of modern logic. In the present flurry of attention given to the semantics of natural language, surprisingly little effort has been focused on the problem of logical inference in natural language and the possibility of its completeness. Even the traditional theory of the syllogism does not give a thorough analysis of the restricted syntax it uses.

My objective is to show how a theory of inference may be formulated for a fragment of English that includes a good deal more than the classical syllogism. The syntax and semantics are made as formal and as explicit as is customary for artificial formal languages. The fragment chosen is not maximal but is restricted severely in order to provide a clear overview of the method without the cluttering details that seem to be an inevitable part of any grammar covering a substantial fragment of a natural language. (Some readers may feel the details given here are too onerous.)

I am especially concerned with quantifier words in both object and subject position, with negation, and with possession. I do not consider propositional attitudes or the modalities of possibility and necessity, although the model-theoretic semantics I use has a standard version to deal with such intensional contexts.

An important point of methodology stressed in earlier publications (Suppes, 1976; Suppes & Macken, 1978; Suppes, 1979) is that the semantic representation of the English sentences in the fragment uses neither quantifiers nor variables, but only constants denoting given sets and relations, and operations on sets and relations.

In the first section, I rapidly sketch the formal framework of generative syntax and model-theoretic semantics, with special attention to extended relation algebras. The second section states the grammar and semantics of the fragment of English considered. The next section is concerned with developing some of the rules of inference. The results given are quite incomplete. The final section raises problems of extension. Classical logic is a poor guide for dealing with inferences involving high-frequency function words such as of, to, a, in, for, with, as, on, at, and by. Indeed, the line between logical and nonlogical inference in English seems to be nonexistent or, if made, highly arbitrary in character—much more so than has been claimed by those critical of the traditional analytico-synthetic tradition.

No theorems on soundness or completeness are considered because of the highly tentative and incomplete character of the rules of inference proposed. However, because of the variable-free semantics used, soundness is easy to establish for the rules given.

I. Generative grammars and their model structures

A couple of examples will illustrate the framework I have in mind and provide an intuitive introduction to those unfamiliar with the concept of semantic trees for context-free grammars.

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Consider the tree for the sentence *Some people do not eat some vegetables* shown in Figure 1 (with the stress on *eat*). On the left of the colon at each node is shown the terminal or nonterminal label. The nonterminal grammatical categories should be obvious: $S =$ sentence, $EQ =$ existential quantifier, $NP =$ noun phrase, $Aux =$ auxiliary, etc. To the right of the colon at a node is shown the denotation of the label if it has one. Thus, in the semantic tree of Figure 1, *people* and its ascendant $NP$ node have the set $P$ of people as denotation; *eat* and its ascendant $TV$ — transitive verb — node have the binary relation $E$ of eating as denotation; *vegetables* and its ascendant $NP$ node have the set $V$ of vegetables as denotation. The $VP$ — verb phrase — node has a denotation composed of set-theoretical operations on $E$ and $V$. Intuitively the denotation is just the set of people who do not eat some vegetables. The notation $\bar{E}$ is for the *converse* of the relation $E$, $\neg \bar{E}$ is the *complement* of the relation $\bar{E}$, and $(\neg \bar{E})^*V$ is the *image* of the set $V$ under the relation $\neg \bar{E}$. I take the root of the semantic tree of a sentence to denote the value $T$ or $F$ in a standard Fregean manner. Most of this notation is standard in elementary set theory (see, e.g., Suppes, 1960). Some subtleties about complementation are discussed below, as is the point about stress.

I give now a quick overview of the relevant formal concepts. First, a structure $G = \langle V, N, P, S \rangle$ is a *phrase-structure grammar* if and only if $V$ and $P$ are finite, nonempty sets; $N$ is a subset of $V$; $S$ is in $N$; and $P \subseteq N^* \times V^+$, where $N^*$ is the set of all finite sequences whose terms are elements of $N$ and $V^+$ is $V^*$ minus the empty sequence. The grammar $G$ is *context-free* if and only if $P \subseteq N \times V^+$. In the usual terminology, $V$ is the vocabulary, $N$ is the nonterminal vocabulary, $S$ is the start symbol of derivations or the label of the root of derivation trees of the grammar, and $P$ is the set of production rules. I assume as known the

![Figure 1](image-url)