1. Introduction

In 1900, at a time when his international prominence as a leading mathematician was just becoming firmly established, DAVID HILBERT (1862–1943) delivered one of the central invited lectures at the Second International Congress of Mathematicians, held in Paris. The lecture bore the title “Mathematical Problems”. At this very significant opportunity HILBERT attempted to “lift the veil” and peer into the development of mathematics of the century that was about to begin (HILBERT 1902, 438). He chose to present a list of twenty-three problems that in his opinion would and should occupy the efforts of mathematicians in the years to come. This famous list has ever since been an object of
mathematical and historical interest. Mathematicians of all specialties and of all countries have taken up its challenges. Solving any item on the list came to be considered a significant mathematical achievement.

The sixth problem of the list deals with the axiomatization of physics. It was suggested by his own recent research on the foundations of geometry; HILBERT proposed "to treat in the same manner [as geometry], by means of axioms, those physical sciences in which mathematics plays an important part (HILBERT 1902, 454)."

This problem differs in an essential way from most others in the list, and its inclusion raises many intriguing questions. In the first place, as formulated by HILBERT, it is more of a general task than a specific mathematical problem. It is far from evident under what conditions this problem may be considered to have been solved. In fact, from reports that have occasionally been written about the current state of research on the twenty-three problems, not only is it hard to decide to what extent this problem has actually been solved, but moreover, one gets the impression that, from among all the problems in the list, this one has received the least attention from mathematicians.¹

From the point of view of HILBERT's own mathematical work, additional historical questions may be asked. Among them are the following: Why was this problem so central for HILBERT that he included it in the list? What contact, if any, had he himself had with this problem during his mathematical career? What was the actual connection between his work on the foundations of geometry and this problem? What efforts, if any, did HILBERT himself direct after 1900 to its solution?

These questions are particularly pressing because of their bearing on the often accepted identification between HILBERT and the formalist approach to the foundations of mathematics. HILBERT's main achievement concerning the foundations of geometry was — according to a widely-held view — to present this mathematical domain as an axiomatic system devoid of any specific intuitive meaning, in which the central concepts (points, lines, planes) could well be replaced by tables, chairs and beer-mugs, on condition that the latter are postulated to satisfy the relations established by the axioms. The whole system of geometry should remain unaffected by such a change. Therefore, it is often said, HILBERT promoted a view of mathematics as an empty formal game, in which inference rules are prescribed in advance, and deductions are drawn, following those rules, from arbitrarily given systems of postulates.² If this was

¹ See, e.g., WIGHTMAN 1976, GNEDENKO 1979.
² Such a view has been put forward by, e.g., the French mathematician JEAN DIEUDONNÉ (1906–1992). In a widely read expository article, DIEUDONNÉ explained the essence of HILBERT's mathematical conceptions by analogy with a game of chess. After explaining that in the latter one does not speak about truths but rather about following correctly a set of stipulated rules, he added (DIEUDONNÉ 1962, 551. Italicics in the original): "Transposons cela en mathématiques, et nous aurons la conception de HILBERT: les mathématiques deviennent un jeu, dont les pièces sont des signes graphiques se distinguant les uns des autres par leur forme."