UNSTEADY HEAT TRANSFER FROM A ROTATING DISK TO FLUIDS WITH LOW PRANDTL NUMBERS

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Summary

An analysis is presented concerning unsteady heat transfer from a rotating disk to a low Prandtl number fluid under the condition of a step change in surface temperature with time. Entire time history results for the surface heat flux are given for Prandtl numbers up to 0.04, and these results are obtained by means of a first-order perturbation about the solution for zero Prandtl number. Steady-state heat transfer predicted by this method agrees almost precisely with exact values.

§ 1. Introduction. In a recent investigation\(^1\) the problem of unsteady heat transfer from a rotating disk was considered under the condition of a step change in surface temperature with time. Unfortunately, the approximate method of analysis which was employed was found to be inapplicable to low Prandtl number fluids (liquid metals), and results were thus presented only for Prandtl numbers indicative of common gases and ordinary liquids. Nevertheless, some qualitative information was obtained concerning low Prandtl numbers, and it was found that the time required to reach steady state (response time) possessed a minimum value near a Prandtl number of one. For decreasing Prandtl number it appeared that the response time would increase quite sharply. This is in contrast with other unsteady forced-convection analyses (Cess and Sparrow\(^1\), and Sparrow and Gregg\(^2\)) which indicate that for flow at a stagnation point or across a flat plate the response of the thermal boundary layer becomes monotonically more rapid with decreasing Prandtl number.

As a result of this rather contrary behavior associated with the rotating disk, the present investigation deals specifically with un-
steady heat transfer from a rotating disk to low Prandtl number fluids. The purpose is to predict the entire time history of the heat transfer process for a step change in surface temperature with time. Heat transfer results are presented for Prandtl numbers of 0, 0.02, and 0.04, which encompass the liquid metal range.

§ 2. Analysis. The physical model is that of a large disk rotating in an infinite and otherwise quiescent fluid, and steady laminar flow of a constant-property fluid with negligible viscous dissipation is assumed. Initially the disk surface and fluid are at the common temperature $T_\infty$, whereupon at $t = 0$ the surface temperature is changed to and maintained at a constant value $T_w$.

The equation expressing conservation of energy for flow about a rotating disk may be written in two-dimensional cylindrical coordinates as

$$\frac{\partial T}{\partial t} + u_r \frac{\partial T}{\partial r} + u_z \frac{\partial T}{\partial z} = \alpha V^2 T,$$

(1)

where $\alpha$ denotes the thermal diffusivity of the fluid, and $u_r$ and $u_z$ are the radial and normal velocity components respectively. It will now be convenient to introduce dimensionless quantities defined as

$$\xi \equiv \frac{z}{\alpha \sqrt{\omega \nu}}, \quad \tau \equiv \frac{\nu \omega}{\alpha} t,$$

$$H \left( \frac{\xi}{Pr} \right) = \frac{u_z}{\sqrt{\omega \nu}}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty},$$

where $\nu$ is the kinematic viscosity of the fluid, $\omega$ the angular velocity of the disk, and $Pr = Pranndtl number = \nu / \alpha$. With these definitions, (1) becomes

$$\frac{\partial \theta}{\partial \tau} + H \left( \frac{\xi}{Pr} \right) \frac{\partial \theta}{\partial \xi} = \frac{\partial^2 \theta}{\partial \xi^2}$$

(2)

and possesses the boundary conditions

$$\theta = 0; \quad \tau = 0,$$

$$\theta = 1; \quad \xi = 0, \quad \tau > 0,$$

$$\theta \to 0; \quad \xi \to \infty.$$

*) One may note that $\xi = Pr \sqrt{\omega \nu}$, where $z \sqrt{\omega \nu}$ is the conventional dimensionless distance employed in rotating disk analyses.