ON THE FUNCTIONAL DEPENDENCE
OF TOWNSEND'S FIRST IONIZATION
COEFFICIENT *)

by A. N. KONTARATOS

STD Research Corporation, Pasadena, California, U.S.A.

Summary
An expression is derived of Townsend's ionization coefficient in terms of
electric field and pressure, which fits the experimental data for air in the
range $E/p = 10$ to $10^3$ Volts/cm $\times$ mm Hg.

§ 1. Introduction. The space charge amplification observed in a
gaseous discharge gap during the development of an avalanche is
ascribed to ionization by electron collisions. An initial electron
drifting in the direction of the applied field will ionize gas molecules
provided that the field is sufficiently high. The secondary electrons
thus formed will also produce further ionization, and the whole
process, being rapidly cumulative, is appropriately termed an
electron avalanche.

Townsend 1) introduced a coefficient $\alpha$ to define the number of
electrons produced over a distance of 1 cm travelled by a single
electron in the direction of the applied field. The total number $n$
of ion pairs produced by one initial electron after the subsequent
avalanche has developed a distance $x$ across the gap is accordingly
given by

\[ n = \exp \alpha x. \] (1)

This coefficient $\alpha$ was shown by Townsend to vary with the
applied electric field $E$ and the pressure as determined by the
relation

\[ \alpha/p = A \exp \left( -\frac{B}{E/p} \right), \] (2)

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where \( A \) and \( B \) are constants. It is impossible, however, to fit a curve of this form to the available experimental data\(^2\)\(^3\)\(^4\) except of course piecewise and then again always within a very limited range.

It is the purpose of this paper to attempt a theoretical determination of a functional dependence of Townsend's ionization coefficient that will describe the experimental data in the available range.

§ 2. Electron statistics. A theoretical calculation of \( \alpha \) must include consideration of the electronic energy distribution function \( F(\omega) \), the probability \( P(\omega) \) that an electron with energy \( \omega \) will ionize a gas atom or molecule by collision, and the average number of collisions \( N(\omega) \) per electron in unit length parallel to the direction of the applied field. Then the ionization coefficient can be obtained from

\[
\alpha = \int_{V_i}^{\infty} N(\omega) P(\omega) F(\omega) \, d\omega,
\]

where \( V_i \) is the ionization potential of the gas atom or molecule.

If it is assumed that the ionization reaction is in equilibrium at the electron temperature, then the non-equilibrium distribution of the electron energies is of the utmost importance in this derivation. This non-equilibrium distribution is the result of the elevation of the electron temperature by energy gain in the electric field. If the frequency of electron-electron collisions is comparable to that of electron-ion and electron-atom collisions, the electrons will acquire a Maxwellian distribution. However, the effective temperature of this distribution may be considerably higher than the gas temperature since the electron species as a whole gains energy from the electric field. Calculations have been carried out for simplified collision laws by neglecting electron-electron and electron-ion collisions. Such a calculation leads to the Druyvesteyn distribution\(^5\) for hard-sphere collisions. Chapman and Cowling\(^6\) have given the correction for the thermal motion of the atoms and Cahn\(^7\) has computed the electronic distribution function including electron-electron collisions.

One result of this calculation is that the electronic energy distribution is nearly Maxwellian for electronic concentrations above