THEORETICAL SOLUTION OF SIMULTANEOUS
HEAT AND MASS TRANSFER BY FREE
CONVECTION ABOUT A VERTICAL FLAT PLATE

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Abstract
In this paper we present a numerical solution for stationary simultaneous
heat and mass transfer in the laminar boundary layer on a vertical flat plate.
The theory is based on a simple physical model that treats the two buoyancy
effects (originating from temperature and concentration differences) as
mutually independent and additional forces.
Results are given for the case of $Pr = 0.71$, $Sc = 0.63$ and equally
directed body forces. Our experimental data for the same conditions show
close agreement and these seem to be no need for a more complicated theory.

Nomenclature
\begin{align*}
a & : \text{thermal diffusivity} \\
A & : \text{empirical constant} \\
B & : \text{total pressure} \\
c & : \text{transformation constant} \\
C & : \text{mass fraction, } = \text{ratio of partial density to total density} \\
D & : \text{mass diffusivity} \\
f & : \text{dimensionless stream function (7)} \\
g & : \text{acceleration of gravity} \\
Gr_x, Gr_L & : \text{local and mean Grashof number (12)} \\
L & : \text{plate height} \\
M_1, M_2 & : \text{molecular weights} \\
n_1, n_2 & : \text{molar densities} \\
Nu_x, Nu_L & : \text{local and mean Nusselt number (11)} \\
p & : \text{vapour pressure} \\
Pr & : \text{Prandtl number } = \nu / \alpha \\
Sc & : \text{Schmidt number } = \nu / D \\
Sh_x, Sh_L & : \text{local and mean Sherwood number (11)} \\
T & : \text{fluid temperature}
\end{align*}
§ 1. Introduction

The problem of simultaneous heat and mass transfer by free convection about a vertical flat plate with uniform surface temperature and concentration can be solved with the integral method used by Somers [1] and Wilcox [2]. This solution requires certain assumptions concerning the boundary layer profiles of velocity, temperature, and concentration. Mathers et al. [3] gave a more exact solution by integrating the boundary layer equations without any a priori assumption. Their solution, however, is only valid for large $Pr$ and $Sc$ numbers, since they neglected the inertia terms in the momentum equation.

In the present paper we obtain a numerical solution of the boundary layer equations for $Pr = 0.71$ and $Sc = 0.63$ without neglecting these inertia terms. This treatment is closely related to that described by Cheesewright [4] for pure heat transfer. Our theoretical solution agrees well with our experiments on heat transfer with simultaneous water evaporation into air (Bottemanne [5]).

In our calculations and experiments we only considered body