THERMAL STRESSES IN A SOLID CONTAINING PARALLEL CIRCULAR CRACKS

M. K. KASSIR and A. BREGMAN

Dept. of Civil Engineering,
The City College of the City University of New York,
(N.Y. 10031) U.S.A.

Abstract
This paper presents an analysis of the steady-state thermal stresses and displacements in an infinite elastic medium containing two or more parallel coaxial circular cracks. A "perturbation" technique is employed to reduce the problem of finding the temperature and the induced stresses to integral equations of Fredholm type which may be solved by numerical means or iterations. Two types of prescribed thermal conditions are considered. The first is concerned with a uniform flow of heat disturbed by insulated cracks and the second deals with stress-free cracks whose surfaces are exposed to identical amounts of heat. The details of the analysis are illustrated by considering the case of two cracks symmetrically located about the mid plane of the solid. When the cracks are of equal radii, iterative solutions of the governing integral equations are derived and used to determine expressions for the stress-intensity factors (opening and edge-sliding modes), displacements of crack surfaces and other quantities of physical interest which are valid for widely spaced cracks.

§ 1. Introduction
During the past decade considerable attention has been devoted to the calculation of thermal stresses and displacements around cracks embedded in elastic solids of infinite extent. The basic three-dimensional geometries solved include the Sack – Sneddon penny-shaped crack and the more general crack of an elliptical form [1, 2, 3]. Thermal stresses in the vicinity of thin rigid elliptical inclusions disturbing a uniform flow of heat are considered in [4]. The list dealing with two dimensional sheets weakened by cracks, inclusions or cavities of various shapes is much more extensive and no at-
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tempt is made to cite some of the pertinent references. Such knowledge is useful in assessing the fracture and fatigue due to thermal cycling [5, 6].

In this study similar problems are investigated for thick elastic solids containing parallel penny-shaped cracks and transmitting heat under steady-state conditions. Collins' [7] method is extended to determine the induced variation of stress and deformation when a simple flow of heat is disturbed by deviation round the edges of the cracks (problem I) and when the surfaces of the cracks are heated – or cooled – by uniform temperature (problem II). The latter situation would arise physically if every point of the elastic solid were to be heated (cooled) uniformly except the areas occupied by the cracks. Then, by the usual superposition principle, the non-trivial thermal stress boundary problem corresponds to that considered in problem II. In both problems it is assumed that there is no penetration between crack surfaces. In the case that the crack surfaces will close then no disturbance will occur. This situation however will not be considered.

The analysis involves the representation of the temperature and the induced displacement at any point of the solid as the sum of several terms each of which is an integral representation involving an unknown function due to one crack in an otherwise continuous medium. Making use of the boundary conditions on the plane containing the cracks, the problem of determining the unknown functions is reduced to that of solving linear Fredholm integral equations of second kind which are valid for any distance between the cracks. However, when the cracks are widely spaced, an iterative solution of the governing integral equations may be readily obtained. Such an approach was employed by Love [8] in discussing the electrostatic potential due to a pair of parallel disks in a field of unbounded extent. See also [9].

The method of solution is illustrated in detail by considering two identical cracks situated at large distance (2d) apart compared to their respective radii (a). Iterative solutions of the governing Fredholm equations in terms of power series in (a/d) are obtained and used to derive various expressions of physical interest. Of particular significance is the result that two different types of stress-intensity factors, $k_1$ (opening mode) and $k_2$ (edge-sliding mode) are operative along the crack border for both types of thermal con-