NOTE ON AN AIRFOIL
IN A SLIGHTLY NON-UNIFORM STREAM

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Summary
An investigation is presented of the modifications that must be made to
two-dimensional calculations of the flow past an airfoil when the flow takes
place in a symmetric channel with slightly non-parallel walls.

Frequently the flow in which an airfoil or hydrofoil is immersed
does not have a constant uniform velocity as is usually assumed.
Instead the mean flow past the airfoil can be accelerated by
neighboring surfaces or other bodies. An example of such a non-
uniform flow would be that in the working section of a two-di-
Mensional wind tunnel or water tunnel in which the walls diverged
or converged slightly. In a different context the mean axial flow
through rotor or stator rows of turbomachines is often speeded up
or slowed down by the contraction of the hub and case. It is often
the case that the concept of an isolated airfoil or airfoils in a
cascade arrangement is used as a part of the analysis of such flows;
furthermore, it is usually assumed that the flow past the airfoil or
through the cascade is a plane two-dimensional flow with a uniform
velocity normal to the cascade axis. The purpose of the present note
is to examine the modifications that must be made to two-di-
Mensional calculations when the flow takes place in a symmetric
channel with slightly non-parallel walls.

The gap between the walls will be assumed to vary slowly with
distance. The velocity will of course vary from one wall to the other.
It may be shown, however, that if the channel is symmetric, the
slope of the wall is small compared to unity, and the changes in
slope are similarly small, the variations in velocity from one wall
to another are of second order of smallness in these quantities. For
simplicity we will consider only the flow in an infinitesimal stream tube containing the plane of symmetry. The continuity equation for an incompressible fluid in Cartesian coordinates is

\[ \frac{\partial}{\partial x} (uh) + \frac{\partial}{\partial y} (vh) = 0, \]  

(1-a)

where \( h \) is a parameter expressing the divergence of the channel walls. The irrotational condition is

\[ \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0. \]  

(1-b)

Both of these equations*) are exact on the plane of symmetry. However, the velocity components \( u, v \) may also be regarded as the average velocity across a finite gap between the walls provided that the slope is sufficiently small. In this case (1-a) is still exact but the irrotational condition is approximate to the order of \((\partial h/\partial x)^2\) etc. In what follows it is simplest to regard these equations as being satisfied on the plane of symmetry where they are exact.

A stream function and velocity potential may be introduced in the usual way

\[ u = \frac{\partial \psi}{\partial x} = \frac{1}{h} \frac{\partial \phi}{\partial y}, \]

\[ v = \frac{\partial \psi}{\partial y} = -\frac{1}{h} \frac{\partial \phi}{\partial x}, \]  

giving the equations

\[ \psi_{xx} + \psi_{yy} + \frac{h_x}{h} \psi_x = 0, \]

\[ \phi_{xx} + \phi_{yy} - \frac{h_x}{h} \phi_x = 0, \]  

(3)

to be satisfied for the velocity potential and stream function respectively. The subscripts denote differentiation with respect to the variable indicated and it is assumed for simplicity that \( h \) varies only