FLUCTUATING FLOW PAST A CYLINDER AT HIGH FREQUENCIES

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Abstract. The flow in a laminar boundary layer for an arbitrary periodic main stream is considered at high frequencies when the fluid is incompressible. The analysis, incorporating length scales appropriate to the thin “Stokes” layer immediately adjacent to the surface and to the outer “Prandtl” boundary layer, involves expressing the dependent variables as mean parts plus superimposed periodic parts and expanding these in inverse powers of the frequency parameter in the two layers. Thus earlier approaches based on physical arguments are placed in the context of a systematic mathematical expansion scheme which is itself formulated for more general main stream velocities than hitherto. Expressions for skin friction and heat transfer are obtained and briefly discussed.

1. Introduction

Lin [2] considered the flow in a laminar, incompressible boundary layer for which the external velocity distribution \( U_e(x, t) \) consists of a steady part and a superimposed periodic oscillation of the form

\[
U_e(x, t) = \bar{U}_e(x) + U'_e(x, t),
\]

where \( x \) is distance parallel to the surface, \( t \) is the time and the bar denotes the average value taken over a period. At high frequencies he developed a method of solution, based on physical arguments to deduce the most important terms in the equation, without the necessity of linearizing with respect to the amplitude of oscillation. Later Pedley [3] considered a similar problem, at both small and large frequencies, for the important case where \( U_e(x, t) \propto x^n \) (\( n \) constant). Similarity variables are then available and by non-dimensionalizing in appropriate ways Pedley was able to obtain series solutions in powers and inverse powers of the frequency parameter. For some flow properties, for example the skin friction when \( n = 0 \), Pedley’s solutions gave a satisfactory coverage over the whole frequency range.

In the present paper we consider again the more general main stream (1) and, for large values of the frequency parameter, place the
Lin [2] approach in the context of formal expansions of the Pedley type. Thus each flow quantity is expressed as a mean part and a superimposed oscillating part like (1), but each part is assumed to take the form of series valid in two layers, the thinner Stokes layer whose thickness is $O(v/\omega)^{1/2}$, where $v$ is the viscosity and $\omega$ the frequency and the much thicker Prandtl layer associated with the velocity $\bar{U}_e(x)$. In this way it is seen how Lin's solution fits into the formal mathematical analysis. An advantage with the present method is that higher order terms can be straightforwardly, if often tediously, obtained. Further it can be adapted to more complex phenomena, such as periodic boundary layers in compressible fluids, and so we also discuss here the effect of the oscillation on small temperature differences in the flow field.

In section 2 the problem is formulated and the variables are scaled in two different ways for the two layers. The equations satisfied by the coefficients in the assumed expansions are given in section 3 and the stream function obtained in section 4 as expressions for mean and fluctuating parts in each layer. Using this solution in section 5 the temperature distribution is calculated along similar lines. The results, mainly based on expressions for the skin friction and heat transfer are discussed in section 6. At even moderate values of the fluctuation amplitude the skin friction may easily be dominated by its unsteady part at sufficiently high frequencies, a result which emerged from Lin's work, whereas there is relatively little effect on the temperature, which in this limit is governed by its steady part. An opportunity is taken to correct an error in the expression given by Pedley [3] for the heat transfer at high frequencies. An appendix is included to explain how the method can cope with higher harmonic terms present in the external fluctuating velocity field.

2. Formulation of the problem

The boundary-layer equation for two-dimensional unsteady, laminar flow of an incompressible fluid is

$$\begin{align*}
\left(\frac{\partial}{\partial t} + U_0 \frac{\partial}{\partial x}\right) \frac{\partial \Psi}{\partial y} = \frac{\partial U_e}{\partial t} + U_e \frac{\partial U_e}{\partial x} + v \frac{\partial^3 \Psi}{\partial y^3},
\end{align*}$$

(2)

where $(x, y)$ are the coordinates along and normal to the surface, $(\partial \Psi/\partial y, -\partial \Psi/\partial x)$ the corresponding velocity components $(u, v)$ in terms of the stream function $\Psi$, $\rho$ the density of the fluid and the convective