FINITE CYLINDRICAL DEFORMATION OF
A CORD-REINFORCED RUBBER BELT

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Abstract
The deformations and stresses in an initially straight belt, bent and sheared around a rigid cylindrical drum, are theoretically investigated. The belt is made of a Mooney-type material and reinforced by a cord layer, assumed thin and inextensible. Three types of solutions are obtained, where the actual type is shown to depend on the conditions of the initial application of the belt to the drum.

Nomenclature

- Cartesian material coordinates of the undeformed body
- Arc angle of the deformed body
- Constants in the Mooney strain energy function
- Radial-dependent functions defined by (7)
- Covariant and contravariant metric tensor components of the undeformed body
- Covariant and contravariant metric tensor components of the deformed body
- Relative shear displacement of the cord
- Strain invariants
- Axial extension ratio
- Applied torque per unit width
- Coefficient of friction between belt and drum
- Pressure function
- Inner, cord, resp. outer radii in the deformed state
- Cylindrical spatial coordinates of the deformed body
- Physical stress components of the deformed body
- Stress tensor components of the deformed body
- Cord tension per unit width
- Strain energy function

Subscript \( \partial \) denotes partial differentiation with respect to \( q \).
§ 1. Introduction

The present problem deals with the combined bending and angular shear deformation of a reinforced rectangular block of a rubberlike material around a cylindrical drum. The body is treated as consisting of two incompressible, isotropic, elastic blocks, whose common boundary is a thin, inextensible cord, thus imposing a restraint on the boundary conditions. Stresses and deformations are calculated for the cord as well as for the rubber material. The analysis follows the general theory for finite elastic deformations described by Green and Zerna [1].

According to the analysis by Ericksen [2] no universal solution exists for this type of deformation, unless the tangential shear is zero. However, by introducing the Mooney strain energy function as a constitutive relation, the equilibrium equations are simplified and exact solutions can be found. Similar results were obtained by Hill [3].

The accuracy of the Mooney equation in describing the behaviour of rubber is discussed by Treloar [4] and others.

The problem considered has a technical application in the construction of conveyor belts, made of two relatively thick sheets of rubber or rubber-like material surrounding a densely web-reinforced middle layer. It is observed that the traction force in the belt, carried mainly by the reinforcement, rises at the end drums. Gerber [5] used a simple model to calculate this increment.

§ 2. Analysis

Pre-requisites.

Equilibrium conditions for the belt section considered are investigated under the following assumptions:

Planes parallel to the cord deform into concentric circles around the drum. The deformation is uniform along the axis.

The axial extension is uniform, implying a plane strain case.

The reinforcing cord is thin and inextensible.

The material is incompressible and obeys the Mooney equation of strain energy: \[ W = C_1(I_1 - 3) + C_2(I_2 - 3). \]

Volume and inertia forces need not be taken into account.

Deformation

If cylindrical coordinates \((r, \theta, z)\) are chosen to represent the position in the deformed state of the material points initially at \((a_1, a_2, a_3)\), the