ON MODEL CONSTANTS AND SECOND ORDER CLOSURE FOR CURVED SHEAR FLOWS

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Abstract
The pressure-strain correlation model of Hanjalic and Launder is extended to rotating curved shear flows. Stationary plane flow data together with the requirement that the critical Richardson number should be predicted correctly are used to determine the model constants. The value of the constants are found to be the same as those determined by Hanjalic and Launder through computer optimization. It is also found that the pressure-strain correlation model of Hanjalic and Launder has negligible effects on the shear stress relation for curved shear flows compared to the simple "return to isotropy" model of Rotta. In view of this, significant improvement on second order closure of the mean flow equations for curved shear flows cannot be obtained by improving the simple Rotta model.

Nomenclature

- $a_1, a_2, a_3$: Constants defined in (11a...c)
- $A_1...A_6$: Constants defined in (11d...i)
- $b_{ij}$: Reynolds stress matrix defined in (12)-(15)
- $c_1$: Model constant introduced by Rotta [2] and defined in (6)
- $c_2$: Model constant introduced by Rotta [2] and also in (3)
- $c_p, c_3$: Model constants introduced in (3)
- $F_c$: Curvature parameter defined in (11j)
- $F_R$: Rotation parameter defined in (11k)
- $k$: Longitudinal surface curvature
- $l_1$: Length scale introduced in (3)
- $l_b$: Mixing length for corner flow defined by Gessner and Po [13]
- $p = \frac{p'}{\rho}$: Fluctuating pressure
- $q^2 = u^2 + v^2 + w^2$: Twice the turbulence kinetic energy
- $R_i$: Gradient Richardson number for curved shear flows
- $R_i_{cr}$: Critical gradient Richardson number
§ 1. Introduction

Closure of the Reynolds-stress equations depends on the modeling of the dissipation, diffusion and energy redistribution terms in the equations. Numerous models have been proposed for these three terms, however, none has created more disagreement than the models proposed for the energy redistribution or pressure-strain correlation terms [1]. A simple model was proposed by Rotta [2] in which the term was partitioned into a "return to isotropy" and a "rapid distortion" part. With some simplifications, Rotta's model could be conveniently written as

$$-rac{c_1}{T} \left( \overline{u_i u_j} - \delta_{ij} \frac{q^2}{3} \right) + c_2 q^2 \left( \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right)$$

where $T$ is a time scale for the energy containing eddies and $c_1, c_2$ are model constants to be determined. The first term is the "return to isotropy" part of the model and the second term is the "rapid distortion" part. However, the "rapid distortion" part of Rotta's model has been shown to give unsatisfactory results in the case of channel flows and various researchers [3, 4] have since tried to improve on Rotta's model. Most improvements are made in the form of an additional function that is linear in the Reynolds stresses. The conditions of symmetry and incompressibility are used to help determine the additional function while the model constants are determined from two-dimensional stationary plane flow data.

Of the various improved models suggested [3–6], the most widely tested is the model of Hanjalic and Launder [3]. Their model is similar to the one proposed by Naot et al. [4]. Following Rotta [2], Hanjalic and Launder [3] introduced two arbitrary constants, $c_1$ and $c_2$, into their model. Again, $c_1$ is used to associate with the "return to isotropy" part of the model, which