SOLUTION OF AN OPEN REGION PROBLEM AS A LIMITING CASE OF A CLOSED REGION PROBLEM

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Summary

Starting from the solution of the bifurcated parallel plate waveguide, the paper derives the solution of an open ended parallel plate waveguide radiating into space. This is achieved by approaching the limit as the transverse dimension of the large guide becomes indefinitely large. It is shown that the expression for the reflection coefficient in the parallel plate waveguide obtained in this manner is the same as that derived by Marcuvitz, Noble, etc., who use the Wiener-Hopf technique. Application of this procedure to other problems, such as surface wave launching on a dielectric or plasma slab, is suggested.

§ 1. Introduction. The purpose of this paper is to investigate whether or not it is possible to derive the solution of an open region problem as a limiting case of a closed region boundary value problem. The particular geometries under consideration appear in figures 1a and 1b. It is shown that the expression for the reflection coefficient in the parallel plate waveguide of fig. 1b may be obtained as a limiting case of $a, b \to \infty$ in the bifurcated waveguide geometry of fig. 1a.

The investigation of this problem was motivated by a recently published work by Talanov¹) who considers the surface wave excitation problem shown in fig. 2a. Since the Wiener-Hopf factorization of the kernel function associated with this problem turns

¹) The paper was orally presented at the International Conference on Microwaves, Circuit Theory, and Information Theory, Tokyo, Japan, 1964.
out to be difficult*), he suggests that useful information can be gained by studying the inhomogeneous bifurcation problem of fig. 2b. However, no analytical proof is given in his paper to demonstrate that such a limit exists. Neither does he compare his numerical solutions for large transverse dimensions of the closed waveguide to the exact solution, since, as pointed out earlier, the exact solution of this problem is intractable in a convenient form.

*) Although the formal Wiener-Hopf factorization is always possible in terms of an integral representation, the difficulty lies in the fact that the general form may be quite involved. The general factorization procedure has been used by Angulo and Chang2) for the surface wave launching problem and the reader is referred to the above work for further details on this approach.