FIELD ANALYSIS AND COMPLEX RESONANCE FREQUENCY OF THE QUASI-TE_{0,1,1}-MODE IN AN INHOMOGENEOUSLY FILLED RESONATOR WITH LOSSES

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Abstract
The object of the present investigation is the determination of the field intensity and the resonance frequency of the quasi-TE_{0,1,1}-mode (with rotational symmetry) in an inhomogeneously filled resonator. The resonator is filled with a number of concentrically located tubes consisting of different materials, which may be lossy. The top and the bottom walls of the resonator are electrically perfectly conducting; at the side wall an impedance boundary condition is employed.

Numerical results for a case of practical interest in magnetic resonance experiments are obtained.

§ 1. Introduction
The object of the present investigation is the determination of the field intensity and the resonance frequency of the quasi-TE_{0,1,1}-mode (with rotational symmetry) in the inhomogeneously filled resonator shown in Fig. 1. To locate a point in the configuration, a circularly cylindrical coordinate system \( r, \theta, z \) with \( 0 < \theta < 2\pi \) is introduced. The \( z \)-axis of this coordinate system coincides with the axis of symmetry of the resonator. The resonator is filled with concentrically located tubes of different (lossy) dielectric materials. In view of the application of the resonator in magnetic resonance experiments, the number of dielectric tubes is chosen as follows:
(a) Two concentric quartz tubes are located at $R_3 < r < R_4$ and $R_5 < r < R_6$, and have a permittivity $\varepsilon_1$; they form a dewar.

(b) A sample tube is located at $R_1 < r < R_2$, it is made of pyrex and has a permittivity $\varepsilon_2$. The sample material to be investigated is located at $0 < r < R_1$ and has a permittivity $\varepsilon_3$. The remaining domains have the permittivity of vacuum $\varepsilon_0$. The permeability is taken to be $\mu_0$ throughout the configuration. To take into account the dielectric losses in the tubes as well as in the sample material, $\varepsilon_1$, $\varepsilon_2$ and $\varepsilon_3$ are taken to be complex. The top and the bottom walls of the resonator, located at $z = H$ and $z = 0$ respectively, are electrically perfectly conducting; at the side wall $r = R_7$ an impedance boundary condition is used.

§ 2. Formulation of the problem

We shall investigate the free oscillations of the resonator under consideration. Then, no sources are present and each of the electromagnetic field components can be written as

\[ E(r, t) = \operatorname{Re}\{E(r; \omega) \exp(i\omega t)\}, \]
\[ H(r, t) = \operatorname{Re}\{H(r; \omega) \exp(i\omega t)\}, \]

where

\[ \omega = \omega' + i\omega'' \quad \text{with} \quad \omega' > 0 \quad \text{and} \quad \omega''. > 0. \]

The choice of the complex angular frequency is in accordance with the presence of lossy material in the resonator which causes, sources being absent, the amplitude of the free oscillations to