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A PROBLEM-ORIENTED MATHEMATICAL OPTIMIZATION COURSE

Summary. A one-semester applied mathematical optimization course has been developed at the University of Wisconsin-Green Bay. The problems for the course were derived from the current research literature in environmental sciences. The use of computer subroutines for problem-solving is a key feature of the course.

1. INTRODUCTION

At the present time the field of mathematical optimization is a lively one. There are numerous applications to problems in water resources, pollution control, waste management, resource management, management science, health care, engineering and other areas. A cursory glance through journals publishing articles in these fields reveals this variety and abundance of applications. In addition, theoretical aspects of mathematical optimization are receiving increasing attention. Existence problems and the design of efficient computer algorithms for calculating optimal solutions must be confronted and solved. Dantzig and Eaves [1] estimate that one-fourth of all current scientific computation involves optimization.

Despite this flurry of activity it appears that many undergraduate mathematics students do not gain a significant exposure to optimization in current curricula. It is obvious that students with career goals in environmental sciences, engineering, management sciences, and systems analysis who are not mathematics majors can benefit from a knowledge of optimization. Dantzig and Eaves [2] suggest that new courses should be developed to familiarize students with mathematical optimization. Wolfe [3] points out the need for the student in optimization to test models by doing numerical experiments and that in all but trivial cases this cannot be done without a computer. For the student of optimization he suggests

aptly designed tasks which require his intimate acquaintance with the model's operation will motivate and teach him in a way lectures and reading cannot.

We fully concur with these opinions. It is the purpose of this article to describe a one-semester junior and senior level course in applied mathematical optimization for undergraduates which has been developed and taught by us at the University of Wisconsin-Green Bay and which we feel meets the needs described by Dantzig, Eaves, and Wolfe. We have offered the course twice and
it has been enthusiastically received by the students. The positive response from students in various disciplines has been one of the significant results of this classroom experiment.

2. THE APPLIED MATHEMATICAL OPTIMIZATION COURSE

The following objectives were formulated to guide the development of the Applied Mathematical Optimization course. Students will:

(i) learn mathematical concepts and ideas needed in the study of mathematical optimization,

(ii) learn the theory of several optimization techniques,

(iii) learn to apply optimization techniques to 'real-life' problems – the process of mathematical modeling,

(iv) learn to use computer subroutines for solving optimization problems.

The class meets three times each week, with two periods devoted to the fulfillment of the first two objectives and one period to the fulfillment of the last two objectives.

The prerequisites for the course include sufficient calculus to insure an introduction to partial differentiation. This topic serves as a background when developing the classical optimization of functions of several variables. Matrix algebra and an introduction to vector spaces is obviously desirable for mathematical optimization, particularly linear programming. During the first two offerings of the course the essentials of matrix algebra and vector spaces were developed in the first part of the semester. However, in future offerings, we will include an elementary linear algebra course as one of the prerequisites.

We do not require computer programming as a prerequisite even though the computer is used extensively in solving the problems which are the key feature of the course. Many optimization subroutines are available as software or are presented in texts. The text by Converse [4] is particularly useful because it provides computer subroutines for optimization techniques. These stored subroutines can be conveniently used by the students, provided carefully prepared instructions are given to the students. It is not one of the purposes of the course to teach students to write computer programs to solve optimization problems. Students with no experience in computer science have found that they are at a small disadvantage in comparison with those with experience because inevitably something occurs which is not covered in even the best set of instructions, requiring that the student improvise. Those with experience in computer science are better able to do so. This liability is offset by reports from some students that the experience in this course has demonstrated to them the need to take a course in computer science.