ON DIFFICULTIES WITH FRACTIONS

ABSTRACT. A sample of less successful students (aged 12–15) was tested on the topic of fractions. The test paper presented diagrams, word problems, and computational questions. The analysis was designed to find out the specific difficulties and deficiencies of these children. It is shown that most of them are only able to apply remembered rules to the solution of the problems without knowing whether the rule works, i.e., their understanding is at most ‘instrumental’, but not ‘relational’.

1. INTRODUCTION

Fractions are recognised to be a difficult topic in school-mathematics. Up to now neither new forms of teaching nor new approaches to the arithmetic of fractions have been able to alter this. The most important causes are given as:

- Fractions are used less often in daily life and are less easily described than natural numbers.
- The written form of the fractions is comparatively complicated.
- It is not easy to put the fractions in order of size on the number line.
- For the arithmetic of fractions there exist many rules, and these are more complicated than those for natural numbers. If these rules are introduced too early, there is a danger of their being used mechanically and without thought (cf., e.g., Bigalke and Hasemann, 1978; or Padberg, 1978).

These statements are, however, too general about difficulties, and therefore are not very useful in practical teaching: neither are the specific difficulties of certain groups of pupils taken into account, nor are the difficulties in particular areas of the arithmetic of fractions defined accurately enough. If one attempts this, one is automatically led away from an analysis of content to an analysis of the behaviour of the pupils with certain problems or types of problems.

One possible approach is to attempt the development of optimal or hierarchical learning sequences for certain arithmetical procedures. Uprichard and Phillips (1977), for example, have generated and validated a hypothesized learning hierarchy for the addition of fractions: “The purposes of the study were (1) to develop a learning hierarchy for rational number addition using the intraconcept analysis technique, and (2) to test the validity of the hypothesized hierarchy using the Walbesser method and pattern analysis.” The authors formed ‘sum categories’ and ‘denominator classes’, and on the basis of their examinations they made some suggestions as to the order in
which these categories and classes should be dealt with. They were not able to define finally an optimal sequence; but it became clear that in the development of teaching sequences "both epistemological and psychological factors" must be considered (cf. Uprichard and Phillips 1977, pp. 7 and 15).

Another approach is to attempt to construct hierarchies of understanding. A broadly conceived investigation in this direction was carried out, e.g., in the framework of the project 'Concepts in Secondary Mathematics and Science' (CSMS) of Chelsea College, London University: "The formation of a hierarchy in each topic (e.g., Ratio, Algebra, Fractions) was based on the grouping of items at a particular facility level using the homogeneity coefficient $\Phi^1$. For items to form a group, they had to be linked (at an acceptable level) with harder and easier items and with others of approximately the same facility. The separation of one group from the next hardest was based mainly on the existence of a facility gap but also on the mathematical coherence of the items. When items on a particular test had been formed into groups these were designed 'levels' and each child in the sample tested was assigned to a level of understanding on the basis of his performance. A child was assigned to level 2 for example if he had passed about 2/3 of the items in each of level 1 and 2. The levels were tested for scalability by using a Guttman scalogram analysis and if the percentage of children who had error type responses (i.e., success on a harder group did not mean success on all easier groups) was above 7 per cent, the chain of groups was reconsidered" (cf. Hart, 1979, p. 88).

We report on the first results of an investigation in the Osnabriick area which was partly suggested by the CSMS project; some of the items in our test are translations of the English items. However, deviating from the English investigation, we have only attempted to define some examples of deficits which some pupils have in certain areas of fractions, and to make suggestions as to how these deficiencies could perhaps be compensated. For this an analysis of the mistakes we found in the pupils' answers was necessary. Tables of mistakes and deficiencies in the topic of fractions, as well as some suggestions for promoting improvement, are to be found in the German literature in Schlaak (1968) and Jaumann (1978) among others. But we did not, like Schlaak, examine in detail the different mistakes of each pupil. Instead, we laid the stress of the investigation on how far a pupil had understood the concepts or rules which he used. To this end a concept of 'understanding' is elucidated in the 4th Section, with the help of which the present results can be interpreted. It can be seen that many of the poorer scholars do not get further than an 'instrumental understanding': they use in particular the rules mechanically, i.e., without understanding why this rule is applicable in this particular case. This explains a great many of the mistakes and supplies at the same time hints of improvement.