Some new aspects of the slow flow of a viscous fluid through an axisymmetric duct expansion or contraction. I — Numerical part

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Abstract. The three-dimensional flow in a circular duct with a sudden expansion or contraction is considered, and a general analytical solution for creeping flow presented. The resulting calculations predict the evolution of the main features of the flow when the section ratio is varying; they are compared with some other existing numerical results. Unusual and detailed information about the rates of deformation in the flow are given.

Nomenclature

\( A_k, a_k \) constants of integration (see (13) and (27))

\( D^*, D \) dimensionalized and dimensionless rate-of-deformation tensor

\( D_{rr}, D_{r\theta}, D_{rz} \) components of the rate-of-deformation tensor

\( D_{\theta\theta}, D_{\theta z}, D_{zz} \) components of the rate-of-deformation tensor

\( FF_i, GG_i, HH_i \) constants of integration

\( F_k(r, z), f_k(r, z) \) functions defined by (15) and (27)

\( G_k(r, z), g_k(r, z) \) functions defined by (16) and (27)

\( H_k(r, z) \) functions defined by (14)

\( I \) function defined by (26)

\( \Im(x) \) imaginary part of the complex number \( x \)

\( I_0, I_1 \) Bessel functions

\( J_0, J_1 \) Bessel functions

\( K_0, K_1 \) Bessel functions

\( k_l \) constants of integration

\( L^2 \) partial differential operator

\( l_e \) entry length

\( m \) root of (30)

\( p^*, p \) dimensionalized and dimensionless pressure

\( P_k(r, z), p_k(r, z) \) functions defined by (18) and (27)

\( Q \) volumetric rate

\( \Re(x) \) real part of the complex number \( x \)

\( r, z \) cylindrical coordinates

\( \text{Re} \) Reynolds number

\( T_k(r, z), t_k(r, z) \) functions defined by (17) and (27)

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1. Introduction

The slow flow of viscous fluids through circular pipes with an abrupt change in the diameter of the section, is encountered in many industrial processing where it is one of the most widespread flow geometries: for example, in the extrusion of a fluid from a large tank or pipe into a small orifice. Both situations, sudden contraction or expansion of the area of the conduit, induce the same flow pattern if inertia is negligible.

Most of the theoretical studies concern the case of two-dimensional plane flows, some of them are summarized by Crochet and Bezy [1]; however, some numerical results dealing with the three-dimensional axisymmetric problem are available in the literature: for small or intermediate Reynolds numbers and diameter ratios which are indicated in Table 1, some information has been obtained by means of finite difference techniques (Macagno and Hung [2], Christiansen, Kelsey and Carter [3], Vrentas and Duda [4]) eventually associated with an alternating direction implicit method (Halmos, Boger and Cabelli [5]) or finite element method (Viriyayuthakorn and Caswell [7]) in the case of a viscous newtonian [2] [3] or non-newtonian [5] [6] [7] fluid. The results of these studies remain fragmentary; they do not allow for many cross-checkings and sometimes they are contradictory. However, all of them relate the existence of a vortex cell, in the case of the expansion and also in the case of the contraction, but the shape and the size of this vortex are different from one author to another and few experimental papers are available which are apt to clear up these discrepancies.