ON THE POLYNOMIAL APPROXIMATION OF BOUNDARY-LAYER FLOW PROFILES

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Abstract
The behaviour of the polynomial approximation to the boundary layer velocity profile is investigated. Various orders of polynomials and 4 different schemes of "reasonable" boundary conditions are examined for applicability as approximate solutions to the Blasius flow over a flat plate.

A variational formulation, based upon the local potential is used to obtain the solution. It is found that the best and most consistent results are obtained when a symmetric distribution of auxiliary boundary conditions on the wall and outer edge of the boundary layer is used. The 6th order polynomial of this type, for example, already gives a wall friction factor within 0.5% of the exact solution.

Nomenclature
\(a\) coefficients of polynomial approximations
\(b\) free parameter, to be varied
\(c_f\) non-dimensional wall friction factor, defined in Eq. (11)
\(F\) functional mathematically equivalent to the equations of motion
\(i, j, k\) indices
\(J\) generalized thermodynamic fluxes
\(L\) length of control volume in streamwise direction
\(t\) time
\(U\) freestream velocity
\(u, v\) velocity components in boundary layer, parallel and normal to the wall, respectively
\(X\) generalized thermodynamic forces
\(x, y\) coordinates parallel and perpendicular to the surface, respectively
\(\delta\) boundary layer thickness
\(\mu\) viscosity
\(\nu\) kinematic viscosity
§ 1. Introduction

The Prandtl boundary layer approach to high Reynolds number flow situations has been one of the most fruitful advances of modern fluid dynamics. Originally developed for incompressible pure fluid flow, it has been later generalized to deal with problems of heat and mass transfer in multiphase, multicomponent, compressible fluids, and many further applications [1].

The boundary layer concept permitted a great simplification of the governing Navier-Stokes equations. However even the simplest cases still result in nonlinear differential equations, with no closed form solutions. This fact has encouraged the development of many approximate methods for solving the boundary layer equations.

One of the most common methods of obtaining such approximate solutions is the choice of a simple function to describe the velocity field in the boundary layer, using realistic boundary conditions to force the function to a form as close as possible to the real velocity distribution. Because of their simplicity and ease of manipulation, polynomials have been a favorite choice ever since von-Kármán and Pohlhausen presented their momentum integral method [2]. They have been applied in diverse fields such as heat and mass transfer [3, 4], diffusion in low Reynolds number – high Schmidt number flows [5] and others.

As mentioned by Schlichting [1] and others it seems natural to try and improve the agreement of the approximate results obtained (von-Kármán and Pohlhausen used a fourth-order polynomial) by raising the order of the polynomial. However, efforts in this direction were not especially successful.

The present paper examines the behaviour of higher order polynomial approximations based on various choices of boundary conditions and compatibility requirements. The method used for finding the “best” coefficients for the polynomials is the variational tech-

\[ \tau \quad \text{shear stress} \]
\[ \phi \quad \text{local potential function} \]

\textit{Subscripts}

A  approximate
E  exact
w  wall