EFFECT OF RADIATIVE HEAT TRANSFER ON THE GROWTH AND DECAY OF ACCELERATION WAVES

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Abstract

The propagation of acceleration waves has been studied along the characteristic path by using the characteristics of the governing quasilinear system as the reference coordinate system. It is shown that a linear solution in the characteristic plane can exhibit non-linear behaviour in the physical plane. As an application of the theory the point is determined where the wave will break down, provided the breaking occurs at the wave front. It is investigated as for how the radiative heat transfer effects under optically thin limit will influence the process of steepening or flattening of acceleration waves with planar, cylindrical and spherical symmetry. The critical time is obtained when all the characteristics will pile up at the wave front to form a shock wave. The critical amplitude of the initial disturbance has been determined such that any compressive disturbance with an initial amplitude greater than the critical one always terminates into a shock wave, while an initial amplitude less than the critical one results in a decay of the disturbance. The radiative heat transfer effects delay the formation of a shock wave and has a stabilizing effect in the sense that not all compressive acceleration waves will grow into shock waves. A non-linear steepening and a radiative heat transfer provide a particular answer to the substantial question as for when a shock wave will be formed.

Nomenclature

\( \rho \) density of the gas
\( u \) gas velocity
\( t \) time
\( x \) spacial coordinate
\( p \) gas pressure
\( a_0 \) the Stefan-Boltzmann constant
\( D_R \) the Rossland diffusion coefficient of radiation
\( T \) temperature K
\( U \) a column matrix
\( B \) a column matrix
\( A_0 \) a square matrix of order 3
\( \delta t \) time-derivative operator as observed from the wave front
\( [Z] \) symbol for jump in the enclosed quantity
\( \mathbf{0} \) a null column vector
\( I \) an identity matrix of order 3
\( c \) sound velocity
\( J \) Jacobian of transformation
\( c_v \) specific heat at constant volume
\( \Sigma(t) \) wave front
\( \delta \) dimensionless parameter of wave amplitude
\( \eta \) dimensionless parameter of time
\( \beta \) dimensionless parameter of radiative heat effects
\( \lambda \) dimensionless parameter of initial acceleration
\( W(\eta) \) an integral function
\( \nu \) parameter of symmetry
\( \gamma \) ratio of specific heats
\( \tau \) wave tag
\( \psi \) particle tag

Letter subscripts denote partial derivatives in general.
Exceptions are
\( c \) critical value
\( * \) initial wave label
\( 0 \) state ahead of the wave

\section{1. Introduction}
Recently Ram and Srinivasan \cite{1} have studied the radiative heat transfer effects on the propagation of pressure shocks. The acceleration waves are of particular interest because of the fact that they are a special class of non-linear wave processes which can be treated rigorously by analytical methods. The explicit results of the analysis give some insight into the interaction of various mechanisms participating in the wave propagation. Bürger \cite{2, 3}, Becker \cite{4, 5} and Schmitt \cite{6} studied the problem of growth and decay of acceleration waves in relaxing gases and obtained the critical time for the shock formation. The same problem was earlier studied by Varley \cite{7, 8} for a family of viscoelastic materials and by Coleman and Gurtin \cite{9} for materials with fading memory. Rarity \cite{10} studied the problem of breakdown of characteristic solutions in flows with vibrational relaxation. Perturbation effects on the decay of discontinuous solutions of non-linear first order wave equations were studied by Murray \cite{11}. Bowen and Chen \cite{12} studied the local and global behaviour of acceleration waves. Bowen and Doria \cite{13} studied the effect of diffusion on the growth and decay properties of acceleration waves in a mixture of elastic gases. Recently Wlodarczyk \cite{14} found closed form solutions to the problem of