Asymptotic Triviality of the Møller Operators in Galilei Invariant Quantum Field Theories

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Abstract. Asymptotic triviality of the Møller operators is shown for fermions interacting via nonlocal pair-potentials with a momentum cut-off. Such systems have stable Hamiltonians, defining the time evolution in the Heisenberg representation, and strong mixing properties.


1. Introduction

Recently [1], it has been shown that the Hamiltonian

\[ H = \int \! d^3x \, \Sigma a^*(x) \Sigma a(x) + \int \! d^3q \, d^3p \, d^3q' \, d^3p' \, a_{q,p}^* a_{q',p'}^* \Sigma \tau(q - q', p - p') a_{q',p'} a_{q,p}, \]  

where \( a_{q,p}^* \) (resp. \( a_{q,p} \)) denote the smeared out fermion creation (resp. annihilation) operators*.

\[ a_{q,p}^* := \int \! d^3x \, f(x) \Sigma a^*(x), \quad a_{q,p} := \int \! d^3x \, f(x) \Sigma a(x), \]  

is stable and defines a time evolution \( \tau_t : \mathcal{A}_F \to \mathcal{A}_F \) of the \( C^* \)-algebra \( \mathcal{A}_F \) generated by the fermion creation and annihilation operators, if

\[ \int \! d^3q \, d^3p \, |\tau(q, p)| < \infty. \]  

such that

\[ \frac{d}{dt} \tau_t(a) = i[H, a] \quad \forall a \in \mathcal{A}_F. \]  

\( \tau_t \) is a one-parameter group of automorphisms of \( \mathcal{A}_F \).

* For the testfunctions we will use coherent states

\[ f_{q,p} := \pi^{-3/4} e^{-1/2|q|^2 - iqp}. \]
In an earlier paper [2], the following notion of trivial asymptotic Møller operators was introduced. (In [2] the misleading notion 'asymptotically trivial' Møller operators was used instead of 'trivial asymptotic' Møller operators.) It has been shown [2] that a Galilei invariant system with trivial asymptotic Møller operators is asymptotically Abelian, hyperclustering, and strongly mixing.

DEFINITION (1.1). Denote by $H_n$ the restriction of the Hamiltonian $H$ in the Fock representation* $\Pi_{\Phi_\beta}(\mathcal{A}_F)$ to the $n$-particle sector; Let $H_{0n}$ denote the free Hamiltonian of the $n$th particle, thus $[H_{n-1}, H_{0n}] = 0$. Define $V_n$ and $V_n(t)$ by:

$$H_n := H_{n-1} + H_{0n} + V_n,$$

$$V_n(t) = e^{iH_{0n}t}V_n e^{-iH_{0n}t}. \quad (5)$$

The asymptotic Møller operators are defined by

$$\Omega_{\pm}^{\text{asy}} := \lim_{t \to \pm \infty} e^{-it[H_{n-1} + H_{0n} + V_n(t)]} e^{it[H_{n-1} + H_{0n}]} \quad (6)$$

We say $(\tau, \mathcal{A}_F, \phi_\infty)$ has trivial asymptotic Møller operators if

$$\Omega_{\pm}^{\text{asy}} = 1 \quad (7)$$

Remark (1.2). Note that for our definition of the asymptotic Møller operators, the existence of the usual (multi-channel) Møller operators is not necessary (see [3]).

2. Interacting Fermion Systems with Trivial Asymptotic Møller Operators

Denote by $H_n$ the restriction of the (formal) Hamiltonian $H$ in $\Pi_{\Phi_\beta}(\mathcal{A}_F)$ to the $n$-particle sector; $V_n = \sum_{n=1}^{n-1} V_{n'}$:

$$H_n := H_{n-1} + H_{0n} + V_n,$$

$$= H_{0R} + H_{rel} + H_{0n} + V_n, \quad (8)$$

where we have split the $n - 1$ particle Hamiltonian $H_{n-1}$ into the operator $H_{0R}$ generating the free motion of the center of mass and the operator $H_{rel}$ generating the relative motion of the $n - 1$ particles.

LEMMA (2.1). $(\tau, \mathcal{A}_F, \phi_\infty)$ has trivial asymptotic Møller operators if, for $t \to \infty$,

$$|t| \sup_{i \in [0,1]} \|V_n e^{it(1-\lambda)H_{0R} + it(1-\lambda)H_{rel} - it\lambda H_{\phi_{\beta}(\pi)}}\| \to 0. \quad (9)$$

for all $\pi^{(n)} \in \mathcal{D}_n, \mathcal{D}_n$ a total set in the $n$-particle Hilbert space $\mathcal{H}_n = L^2(\mathbb{R}^{3n})$.

Proof. Asymptotic triviality of the Møller operators is equivalent to

$$\lim_{t \to \pm \infty} \|e^{it\lambda H_{0n}} - e^{itH_{0n} + H_{n-1} + V_n(t)}\| = 0. \quad (10)$$

* The thermal representation $\Pi_{\phi_\beta}$ associated with the thermal state $\phi_\beta$ via the GNS construction gives just the usual Fock representation for $\beta = \infty$. See [2].