Finite-Dimensional Representations of
$U_q(osp(1/2n))$ and its Connection
with Quantum $so(2n + 1)$

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Abstract. It is shown that the quantum supergroup $U_q(osp(1/2n))$ is essentially isomorphic to the
quantum group $U_q(so(2n + 1))$ restricted to tensorial representations. This renders it straightforward
to classify all the finite-dimensional irreducible representations of $U_q(osp(1/2n))$ at generic $q$. In
particular, it is proved that at generic $q$, every-dimensional irrep of this quantum supergroup is a
deformation of an $osp(1/2n)$ irrep, and all the finite-dimensional representations are completely re-
ducible.

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1. Introduction

The Lie superalgebra $osp(1/2n)$ markedly differs from all the other Lie superalgebras
[1, 2]. While indecomposable but nonirreducible finite-dimensional representations
exist for all the other Lie superalgebras, finite-dimensional representations of
$osp(1/2n)$ are always completely reducible. This makes the representation theory of
$osp(1/2n)$ rather similar to that of ordinary Lie algebras, especially $so(2n + 1)$ [2].
In fact, it was shown by Rittenberg and Scheunert [2] that there is a one-to-one
correspondence between the finite-dimensional representations of $osp(1/2n)$ and the
nonspinorial representations of $so(2n + 1)$. The Clebsch–Gordan decompositions of
tensor products of representations with respect to the two algebras are the same, and
even their centers are identical. However, a concrete explanation for this intriguing
connection between $osp(1/2n)$ and $so(2n + 1)$ was not found within the theory of Lie
algebras and superalgebras, although the simplicity of the representation theory of
$osp(1/2n)$ may attribute, in a broad sense, to the fact that this superalgebra, like all
the ordinary finite-dimensional Lie algebras, has a symmetrizable Cartan matrix.

One of the main aims of this Letter is to explain this connection by exploring
these algebras, respectively. In particular, we will show that the quantum super-
group $U_q(osp(1/2n))$ in its finite-dimensional representations is isomorphic to
$U_q(so(2n + 1))$ restricted to such a class of finite-dimensional representations that
each irreducible component of a representation has an even positive integer as its last
Dynkin label, i.e. nonspinorial. (This isomorphism extends to certain infinite-dimensional irreps as well, see Theorem 3.3 in Section 3 for detail.) Within these representations, \( U_q(osp(1/2n)) \) reduces to the universal enveloping algebra \( osp(1/2n) \) in the limit \( q \) going to 1, and to that of \( so(2n + 1) \) when \( q \to -1 \). Also, finite-dimensional irreps of \( U_q(osp(1/2n)) \) remain irreducible with respect to the resultant algebras in these limits, hence following the results of reference [2].

Using this isomorphism, we also give a complete classification of the finite-dimensional irreps of \( U_q(osp(1/2n)) \) at generic \( q \). It is well known that the representation theory [8] of quantum groups [3] at generic \( q \) is similar to that of Lie algebras. Therefore, one naturally expects that the representation theory of quantum supergroups at generic \( q \) is also some sort of ‘deformation’ of that of Lie superalgebras, and this is indeed the case for \( U_q(gl(m/n)) \) as demonstrated in references [9] and [10]. The above-mentioned isomorphism between \( U_q(osp(1/2n)) \) and \( U_{-q}(so(2n + 1)) \) enables us to show very easily that this is also true for \( U_q(osp(1/2n)) \): at generic \( q \), every finite-dimensional irreducible representation of this quantum supergroup is a deformation of an irrep of \( osp(1/2n) \) with the same weight space decomposition, and all finite-dimensional representations are completely reducible.

Apart from the results of [2], a possible isomorphism between \( U_q(osp(1/2)) \) and \( U_{-q}(so(3)) \) restricted to certain representations was also indicated by the results of [5], where it was observed that every \( U_q(osp(1/2)) \) invariant integrable statistical mechanics model is equivalent to an appropriate quantum \( su(2) \) model. However, so far as we know, this equivalence was not explained in terms of the algebras involved. Recently, Zachos [11] studied symmetry changes within quantum \( su(2) \) modules at \( q = -1 \), and in an indirect way, shed much light on the problem. Inspired by his work, we obtained the general result on the relationship between \( U_q(osp(1/2n)) \) and \( U_{-q}(so(2n + 1)) \).

The plan of this note is as follows. In Section 2 we present the definition of \( U_q(osp(1/2n)) \) and study its finite-dimensional irreps in relation to those of \( osp(1/2n) \). In Section 3 we analyse the relationship between \( U_q(osp(1/2n)) \) and \( U_{-q}(so(2n + 1)) \), proving the complete reducibility of the finite-dimensional representations of the former, and also providing a simple explanation for the results of [2]. Finally in Section 4, we conclude the Letter with some remarks on possible generalizations of the results obtained to other quantum supergroups.

### 2. Finite-Dimensional Irreps

Quantum supergroups are the one-parameter deformations of the universal enveloping algebras of the basic classical Lie superalgebras, and have been studied quite extensively in recent years [7]. The quantum supergroup \( U_q(osp(1/2n)) \) was dealt with by many authors [4, 6, 12]; here we rephrase its explicit definition given in [6]. Introduce the \( n \)-dimensional vector space \( H^* \) with a basis \( \{ \delta_i \mid i = 1, 2, \ldots, n \} \) and define a bilinear form \( (\cdot, \cdot) \) on \( H^* \) such that

\[
(\delta_i, \delta_j) = -\delta_{ij}, \quad \forall i, j.
\]