TIME SERIES VALUED EXPERIMENTAL DESIGNS: A REVIEW

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Abstract. A review is given of the literature on time-series valued experimental designs. Most of this literature is divided into two categories depending upon the factor status of the time variable. In one category, time is an experimental factor, and in the other it is a non-specific factor and enters the design in the context of replications. Analyses in both the time and frequency domain are reviewed. Signal detection models, Bayesian methods and optimal designs are surveyed. A discussion is also presented of application areas which include field trials and medical experiments. A main theme of the literature is that application of standard F-tests to highly correlated data can be misleading. A bibliography of relevant publications from 1949 onward is presented.

1. Introduction

In many situations where an investigation is repeated over time on physically independent material, and where external conditions can be treated as random, it may be sensible to treat time as a non-specific factor; that is, to consider time in the context of replications. For example, the use of automatic data acquisition equipment may make it possible to obtain many observations on the same treatment combination but with only a small time interval between consecutive observations. An example where such a time series valued experimental design and its concomitant analysis would apply is a process control problem in which it is expensive to change the process parameters, but in which it is possible to make observations in a short period of time for a fixed set of parameters. These observations form a time series characterized by a high degree of correlation among contiguous observations. In other experimental situations time is considered as one of the experimental factors, and not just as part of the replication process. The time series aspect of such designs may exhibit autocorrelation sufficiently high that time series methods are required for an appropriate analysis of time as a specific factor. In many cases where data are collected over time it may not be clear that the white noise assumption is valid; one suspects for the most part these cases are analyzed routinely by ANOVA methods without challenging the independence assumption.

This article is meant to serve two purposes: first, to acquaint the general reader with the
fact if the observations under a treatment in an experimental design form a time series or
the data are collected over time, the classical ANOVA methods for testing the treatment
effects without challenging the independence assumption, may be highly misleading; a
modified analysis is required to adjust for correlation induced biases; and secondly, to give
the researcher interested in this area of statistics access to the current and past literature.
The discussion below surveys the various models for time series valued experimental
designs and reviews the different areas where these designs are used; these areas range
from field trials to clinical experiments.

2. The Factor Status for Time

To initiate the discussions we consider a general two-way analysis of variance model
\[ y_{ij}(t) = \mu + \alpha_i + \beta_j + \epsilon_{ij}(t) \]  
\[ i = 1, \ldots, k; j = 1, \ldots, m; t = 1, \ldots, n_0, \]
where: \( y_{ij}(t) \) is the \( t \)-th observation in the \((i,j)\)-th cell, \( \mu \) is the general effect, \( \alpha_i \) is the \( i \)-th
treatment or row effect, \( \beta_j \) is the \( j \)-th column or block effect, and \( \epsilon_{ij}(t) \) is the error variable.
Assume that the \( \epsilon_{ij}(t) \) are autocorrelated and that they could follow as complex a model as
the multiplicative seasonal ARMA process
\[ \phi(B)\Phi(B')\epsilon_{ij}(t) = \theta(B)\Theta(B')\eta_{ij}(t), \]  
where: \( \phi(B), \theta(B) \) are polynomials of degrees \( p \) and \( q \) in non-negative powers of \( B \) with
zeros outside the unit circle; \( \Phi(B'), \Theta(B') \) are polynomials of degrees \( P \) and \( Q \) in
non-negative powers of \( B' \) with zeros outside the unit circle; \( B \) is the backshift operator;
and \( \eta_{ij}(t) \) are components of white noise series that are independent for all \( i = 1, \ldots, k \) and
\( j = 1, \ldots, m \). For example, if \( p = 1, q = 0, P = 0, Q = 0 \) in (2.2), the errors \( \epsilon_{ij}(t) \) follow the
AR(1) process; that is
\[ \epsilon_{ij}(t) = \phi_1 \epsilon_{ij}(t-1) + \eta_{ij}(t). \]
The two-way ANOVA model (2.1) represents a time series valued experimental design. If
a large number of observations are collected on the same treatment combination but with
only a small time interval between consecutive observations, it may be sensible to treat
time as a non-specific factor. Models similar to (2.1) with time as a non-specific factor have
been studied by a number of authors. Among them, we mention Berndt and Savin (1975,
Rothenberg (1984), Mansour et al. (1985), Pantula and Pollock (1985), and Sutradhar et
al. (1987).

In certain experimental situations where data are collected at a few equally spaced time
points, generally with a relatively large time interval between consecutive observations, it
may be sensible to treat time as a specific factor. The two-way ANOVA model with time as
a specific factor may be expressed as