A REMARK ON QUANTUM CORRECTIONS TO CLASSICAL CONFINEMENT

BO-STURE SKAGERSTAM
Institute of Theoretical Physics, Fack, S-402 20 Göteborg, Sweden

ABSTRACT. We discuss some simple models related to quantum corrections of the MIT-bag in the framework of local field theories in two space-time dimensions. A recent result due to A. Chodos and A. Klein is generalized. We find that the corresponding quantum field theories become free or have an energy spectrum unbounded from below.

I. INTRODUCTION

The idea of classical confinement of color degrees of freedom has turned out, in a very remarkable way, to be very useful in the phenomenological description of strong interaction physics [1]. A dynamical description of the corresponding confinement on the quantum level is, however, at present not available.

Quantum corrections to the classical picture mentioned above have been considered in the literature [2, 3] and in simple models it turns out that it is difficult to retain this fundamental ingredient of classical permanent confinement.

In [2] the following Gaussian interaction, involving a charged scalar field \( \phi(x) \)

\[
H_I(x) = B \left( 1 - \exp \left( - \frac{\phi^+(x) \phi(x)}{\epsilon} \right) \right)
\]  \( (1) \)

was considered and it was argued that the field theory, in two space-time dimensions, becomes trivial or unphysical.

In the present note we point out that the analysis in [2] can be simplified by using a minor generalization [4] of a theorem due to S. Coleman [5] concerning the relation between the sine-Gordon equation and the Thirring model.

It is then more or less straightforward to generalize the results in [2].

II. RENORMALIZATION OF THE INTERACTION

As in [2] we shall consider the interaction (1) in one space and one time dimension. Let us first of all consider a single scalar field \( \phi(x) \) and the interaction

\[
H_I(x) = B \left( 1 - \exp \left( - \frac{\phi^+(x) \phi(x)}{\epsilon} \right) \right)
\]
\[
\exp \left( -\frac{\phi(x)\phi(x)}{2\epsilon} \right) = \int_{-\infty}^{\infty} d\mu(\xi) \exp(i\xi\phi(x)),
\]

where \(\mu(\cdot)\) is the following bounded and positive measure

\[
d\mu(\xi) = \exp\left(-\frac{\xi^2}{2}\right) \sqrt{\frac{\epsilon}{2\pi}} \, d\xi
\]

which is normalized to one.

A normal ordering is now sufficient in order to remove all the ultraviolet divergences in (2). In fact Coleman has shown that \[5\]

\[
\exp(i\xi\phi(x)) = N_m \left( \frac{m^2}{\Delta^2} \right)^{\xi^2/8\pi} \exp(i\xi\phi(x)),
\]

where \(N(\cdot)\) denotes a normal ordering operation defined by a mass parameter \(m\) in a standard fashion \[6\]. We therefore define the normal ordering of the Gaussian interaction by the following expression

\[
\exp\left( -\frac{\phi(x)\phi(x)}{2\epsilon} \right) = \int_{-\infty}^{\infty} d\mu(\xi) \left( \frac{m^2}{\Delta^2} \right)^{\xi^2/8\pi} N_m \exp(i\xi\phi(x)).
\]

The integral in (5) can now formally be carried out and we obtain the following result:

\[
\exp\left( -\frac{\phi(x)\phi(x)}{2\epsilon} \right) = \sqrt{\frac{\epsilon}{\epsilon + \frac{1}{4\pi} \ln \left( \frac{\Delta^2}{m^2} \right)}} \left. \left[ N_m \exp\left( \frac{\phi(x)\phi(x)}{2\epsilon} \right) \right] \right|_{\epsilon + \frac{1}{4\pi} \ln \left( \frac{\Delta^2}{m^2} \right)}. \tag{6}
\]

In the case of a charged scalar field we can in a conventional manner introduce two independent scalar fields, \(\phi_1(x)\) and \(\phi_2(x)\), and the following combination \[7\]

\[
\phi(x) = \frac{\phi_1(x) + i\phi_2(x)}{\sqrt{2}}
\]

and we obtain from the Equation (6), since

\[
\exp\left( -\frac{\phi^\dagger(x)\phi(x)}{\epsilon} \right) = \exp\left( -\frac{\phi_1(x)\phi_1(x)}{2\epsilon} \right) \exp\left( -\frac{\phi_2(x)\phi_2(x)}{2\epsilon} \right)
\]

that

\[
\exp\left( -\frac{\phi^\dagger(x)\phi(x)}{\epsilon} \right) = \frac{\epsilon}{\epsilon + \frac{1}{4\pi} \ln \left( \frac{\Delta^2}{m^2} \right)} N_m \exp\left( -\frac{\phi^\dagger(x)\phi(x)}{2\epsilon} \right). \tag{8}
\]

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