NONLINEAR EQUATIONS FOR FIELDS RADIATED BY PARTICLES

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ABSTRACT. Poincaré-invariant differential equations for scalar and vector lightlike fields radiated by a particle moving in an $n$-dimensional Minkowski space are constructed. A possible description of interacting particles by means of singular solutions of these equations is suggested.

0. INTRODUCTION

Consider a particle, which moves in an $n$-dimensional Minkowski space $M_n$ along a given timelike world line

$$y^\mu = y^\mu(s), \quad \mu = 0, 1, \ldots, n - 1,$$

where $y^\mu(s)$ are known functions of the proper time $s$, i.e.,

$$y^2 \equiv y^0 > 0.$$

The point here means the derivation with respect to $s$, $y^2 \equiv \dot{y}^\mu \dot{y}_\mu$, and we use the timelike metric tensor $g^{00} = -g^{11} = \ldots = -g^{n-1}n-1$.

Assume that this particle radiates a lightlike field. Do there exist field equations which have such radiated fields as solutions? We want to find all such equations which (i) are second-order Poincaré-invariant differential equations and (ii) do not depend on the world line of the particle.

We start with scalar fields and show that such equations do exist and are just the conformal-invariant (Liouville, for $n = 2$) field equations, with the only exception for $n = 4$ where we obtain the d’Alembert equations with a source term. A certain modification which includes the $n = 4$ case is considered. Next we sketch the case of vector fields and conclude with a discussion of the construction of many-particle solutions for the equations obtained, i.e., solutions which should describe nontrivial Poincaré-invariant interactions between particles in analogy with a procedure developed in [2, 3] for the case $n = 2$.

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**The search for all possible invariant equations and arbitrariness of $n$ are the main distinctions between this approach and the stimulating work of L. Bel [1].

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1. **Scalar Fields**

The value of a scalar field \( \varphi(x) \) radiated with a light velocity by a particle, depends only on the behaviour of the particle at the point of intersection of the world line (1) with the backward light cone with a vertex at the point \( x \). This means that we consider only *retarded* fields (for the discussion of other possibilities, see the end of this section). As the world line is timelike, such a point always exists and is unique, i.e., the equation

\[
(x - y(s))^2 = 0
\]

uniquely determines the proper time as a function of the point \( x: s = s(x) \), under the condition that the scalar product

\[
r = (x - y(s(x)))\varphi(s(x))
\]

is nonnegative (notice, that \( r = 0 \) iff the point \( x \) lies on the world line). Further, \( \varphi(x) \) must depend only on scalar products of the vectors \( x - y, \dot{y}, \ddot{y}, \ldots \) with substitution \( s = s(x) \). We start with the case when \( \varphi(x) \) is a function only of the first nontrivial scalar \( r \) and consider the general situation later. So we take

\[
\varphi(x) = \varphi(r).
\]

By differentiation of the identity (3) (after substitution \( s = s(x) \)) we obtain

\[
\partial s = z/r, \quad \partial^2 s = (n - 2)/r,
\]

where \( \partial^\mu = \partial/\partial x_\mu, \partial^2 = \partial^\mu \partial_\mu, z = x - y(s(x)) \) and \( r \) is defined by (4). Notice further that we always differentiate with respect to \( x \) both by explicit and implicit (through \( s \)) dependence on \( x \). Now by (4) and (6) we have

\[
(\partial r)^2 = 2p - 1, \quad \partial^2 r = np/r - (n - 2)/r,
\]

where

\[
p = (x - y(s(x)))\varphi^2(s(x))
\]

and \( (\partial r)^2 = (\partial^\mu r)\partial_\mu r \).

A second-order Poincaré-invariant equation for the scalar field must be of the form

\[
\partial^2 \varphi = F(\varphi, (\partial \varphi)^2).
\]

Due to (5) and (7)