The Explicit Form of the Vertex Operator Fields in Two-Dimensional Quantum SL(2, C)-Invariant Field Theory

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Abstract. The explicit form of the vertex operator fields in two-dimensional quantum sl(2, C)-invariant field theory is found.


This Letter, continuing the previous one [1], should be considered as a Commentary to the papers [2–4].

DEFINITION 1 [2]. The model of the Verma modules over the Lie algebra sl(2, C) is the representation of this algebra in the direct integral

\[ \int V_h \, dh \]

of the Verma modules \( V_h \) over the Lie algebra sl(2, C).

The model admits two realisations.

The first realisation is one of Bernstein, Gelfand and Gelfand (BGG realisation). The model space is the space of all holomorphic functions of two complex variables \( t \) and \( z \), where \( z \) belongs to the complex plane \( \mathbb{C} \) and \( t \) belongs to the universal covering \( \mathbb{C}^* \) of the complex plane without zero \( \mathbb{C}^* = \mathbb{C}\setminus\{0\} \). The generators of the Lie algebra sl(2, C) have the form

\[ L_{-1} = z, \quad L_0 = z \frac{\partial}{\partial z} - t \frac{\partial}{\partial t}, \quad L_1 = z \left( \frac{\partial}{\partial z} \right)^2 - 2t \frac{\partial^2}{\partial t \partial z}. \]

The highest vector \( v_h \) of the weight \( h \) has the form \( t^{-h} \).

The second realisation is one in the Fock space over the Laguerre deformation of the complex disk. The model space is the same. The generators of the Lie algebra sl(2, C) have the form

\[ L_{-1} = z, \quad L_0 = z \frac{\partial}{\partial z} - t \frac{\partial}{\partial t}, \quad L_1 = z \left( \frac{\partial}{\partial z} \right)^2 - 2t \frac{\partial^2}{\partial t \partial z} + t^2 \frac{\partial}{\partial t}. \]
The highest vector $v_h$ of the weight $h$ has the form $t^{-h}F(-h, 2-2h; tz)$, where $F(a, b; u)$ is the degenerate hypergeometric function.

Let us denote $0 = \text{span}(t^{-h})$. The representation of the Lie algebra $\mathfrak{sl}(2, \mathbb{C})$ in the model of the Verma modules over it can be naturally extended to the action of the associative algebra $U(\mathfrak{sl}(2, \mathbb{C})) \ltimes 0$ [3, 4]. The operator $T$, which represents the element $t$ of 0, has the form $T = t - \partial/\partial z$ in the realisation in the Fock space over the Laguerre deformation of the complex disk and

$$T = t \sum_{j \geq 0} (tz)^j \frac{1}{(2u + 3) \cdots (2u + 2j + 1)} \frac{\partial}{\partial z},$$

$$u = t \frac{\partial}{\partial t}$$

in the BGG realisation.

**Definition 2** [3, 4]. The operator $X$ in the model of the Verma modules over the Lie algebra $\mathfrak{sl}(2, \mathbb{C})$ is called the vertex operator of the weight $\mu$ iff

$$(\text{ad}(L_{\mu}) - \mu(n + 1)(-T)^{\mu})X = 0.$$ 

The explicit formulae for the vertex operators in the model of the Verma modules over the Lie algebra $\mathfrak{sl}(2, \mathbb{C})$ were written in [1].

Among the vertex operators of the weight $\mu$, there exists only one (modulo a coefficient) which commutes with all degrees of the operator $T$. Such a vertex operator, which is denoted as $B_{\mu}$, has the following form in the Fock space over the Laguerre deformation of the complex disk [1]

$$B_{\mu} = t^{-\mu}F\left(\mu, 2 + t \frac{\partial}{\partial t}; tz\right),$$

where $F(a, b; u)$ is the normalised degenerate hypergeometric function with the operator parameter $b$

$$F(a, b; u) = \sum_{k \geq 0} \frac{u^k \Gamma(b - a) \Gamma(k + a)}{k! \Gamma(a) \Gamma(k + b)}$$

**Definition 3** [3, 4]. The vertex operator of the weight 1 in the model of the Verma modules over the Lie algebra $\mathfrak{sl}(2, \mathbb{C})$ will be called a connection iff $[V, T] = 1$.

The operators $V_h$, defined as

$$V_h(\varphi(T)L_{-1}^k v_h) = \varphi'(T)L_{-1}^k v_h$$

are the connections.

**Definition 4** [3, 4]. The value of the operator $X$ in the model of the Verma modules over the Lie algebra $\mathfrak{sl}(2, \mathbb{C})$, which commutes with all degrees of the