SUPERSYMMETRIC KAC–MOODY ALGEBRA IN GRADED-CHIRAL MODELS

LING-LIE CHAU
Physics Department, Brookhaven National Laboratory, Upton, NY 11973, U.S.A.

JERZY LUKIERSKI and ZIEMOWIT POPOWICZ
Institut for Theoretical Physics, University of Wroclaw, 50205 Wroclaw, Poland

ABSTRACT. It is shown that the supersymmetric Kac–Moody algebra can be generated from the hidden symmetry in graded-chiral models.

1. INTRODUCTION

In recent years, considerable progress has been made in revealing the integrability properties of many interesting nonlinear systems: the chiral, superchiral models in two dimensions, and the self-dual Yang–Mills equations in four dimensions. They all have parametric Bäcklund transformations and their corresponding linear systems have an infinite number of conservation laws [1]. A recent interesting addition to this list is the existence of the infinite-dimensional Kac–Moody algebra generated by a certain hidden symmetry of these systems [2–5], which presumably is the origin of all these fascinating integrability properties. (We call these integrability properties, although the systems discussed here have not been fully shown to be integrable. This is because these properties have been known to be the trademark of many completely integrable systems, e.g. the Sine–Gordon and KdV equations.)

An example of Kac–Moody algebra [6] occurring in $D = 2$ field theory is of the following form

$$[Q_a^{(m)}, Q_b^{(n)}] = C_{ab}^{c} Q_c^{(m+n)} + C n \delta_{m+n,0} \delta_a, b, -\infty < m, n < \infty, \tag{1.1}$$

where the $C$ term is called the center of the algebra. Because, in field-theoretic models, $C \neq 0$ is a result of quantum effects (e.g., follows from normal ordering of the product of fields in the symmetry generators), the centerless infinite algebra (1.1) (with $C = 0$), also known as a loop algebra [7], will be called the classical Kac–Moody algebra. If we multiply (1.1) by $e^{im\lambda} e^{in\lambda'}$ and sum over $m$ and $n$ from $-\infty$ to $\infty$, we obtain

$$[Q_a(\lambda'), Q_b(\lambda)] = C_{ab}^{c} Q_c(\lambda) \delta(\lambda - \lambda') + C \delta_{ab} \delta(\lambda - \lambda'). \tag{1.2}$$

This is now the current algebra [8, 9] form familiar to physicists and the $C$-term is the Schwinger
term. It had been hoped, in the current-algebra approach to particle physics in the late sixties, that the S-matrix could be obtained via the constraints by the current algebra.

Obtaining the S-matrix is certainly one major goal. If, indeed, it turns out that the constraints on these nonlinear systems by the Kac–Moody algebra of Equation (1.1) are so strong that the S-matrices are determinable, it will be a realization of the hope of the current-algebra spirit.

There is another type of infinite-dimensional algebra that is familiar to physicists, i.e., the Virasoro [10] algebra originating from string theories or, equivalently, the infinitesimal conformal algebra in terms of the improved stress tensor in these conformally invariant theories [11]:

\[ [L_n, L_m] = (n - m)L_{n+m} + C(n) \delta_{m+n,0}, \]  

(1.3)

to which the Fourier sum, like from Equations (1.1) and (1.2), gives,

\[ [\theta(x), \theta(x')] = \delta'(x - x')\theta(x) + C \delta'''(x - x'), \]  

(1.4)

where \( C = \frac{1}{12} n(n^2 - 1) \) for the Thirring model, for example.

Recently, the Kac–Moody algebra (1.1) has been extended by mathematicians [12] to super Kac–Moody algebra. In particular, the classical super Kac–Moody algebra looks as follows

\[ [Q^{(m)}_a, Q^{(n)}_b] = C^{ab}_{cd} Q^{(m+n)}_d, \]  

(1.5a)

\[ [Q^{(m)}_a, F^{(n)}_b] = C^{ab}_{cd} F^{(m+n)}_c, \]  

(1.5b)

\[ [F^{(m)}_a, F^{(n)}_b] = C^{ab}_{cd} Q^{(m+n)}_d. \]  

(1.5c)

The extension of the Virasoro algebra (1.3) has also been done by physicists [13] considering the supersymmetric free string models, i.e., (1.3) becomes

\[ [L_m, L_n] = (m - n)L_{m+n}, \]  

(1.6a)

\[ [L_m, F_r] = \frac{1}{2} m - r)F_{m+n}, \]  

(1.6b)

\[ [F_r, F_s] = 2L_{r+s}. \]  

(1.6c)

So far, only the Kac–Moody algebra of Equation (1.1) without the center has been found for the chiral [2], superchiral fields [3] in two dimensions and the self-dual Yang–Mills in four dimensions [4, 5]. Here we show that the supersymmetric Kac–Moody algebra (1.5) can be generated in the graded-chiral models.

2. GRADED-CHIRAL MODEL AND SUPER KAC–MOODY ALGEBRA

We shall consider here an example of \( D = 2 \) graded-chiral SU(n/m) model, described by the principal