ON A GENERALIZATION OF THE EINSTEIN–CARTAN THEORY
AND THE KLEIN–KALUZA THEORY*

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ABSTRACT. The Klein–Kaluza theory with a nonvanishing torsion is developed. The torsion is associated with the spin and polarization of an electromagnetic field. The electromagnetic polarization is considered as a source of additional components of a torsion connected with a fifth dimension. It is proved that the new effects are $10^{36}$ times bigger than the effects from the Einstein–Cartan theory.

1. INTRODUCTION AND SUMMARY

The aim of this paper is to generalize the Klein–Kaluza theory [1], [2] to a situation with a nonvanishing torsion of the connection. The polarization of an electromagnetic field and spin will be associated with torsion. Our generalization of the Klein–Kaluza theory is analogous to the relation of the Einstein–Cartan theory with the general theory of relativity. The diagram (Figure 1) places the Klein–Kaluza theory with torsion among the above-mentioned theories. A new geometric element in our theory is the torsion in the fifth dimension, the source of which is the electromagnetic polarization $M_{\mu\nu}$.

Roughly speaking, if one says that 'mass curves space-time', 'spin twists it' and 'electric charge curves the fifth dimension' then 'electromagnetic polarization twists the fifth dimension'. Naturally, the fifth dimension is understood as a dimension connected with gauging.

The general plan of the paper is as follows. We introduce on a $\mathcal{P}$-metrized electromagnetic bundle, the connection with nonvanishing torsion. This connection is invariant with respect to transformations of group $U(1)$. In this paper we also assume that a torsion of the connection is horizontal.

Next we construct a form of a scalar curvature for this connection and introduce sources. Then, from a variational principle we obtain equations of fields and interpret them. According to the postulate of geometrization of physical quantities, we shall obtain equations where, on the left-hand side, there will be geometric quantities and on the right-hand side, matter quantities.

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In this way, matter quantities will be sources of geometry. We shall obtain an interpretation of electromagnetic polarization as a torsion related to the fifth dimension. We get equations of gravitation in the Einstein—Cartan theory. On the right-hand side, as source, will be the sum of energy momentum tensors of an electromagnetic field with the polarization of matter in the form given by W. Israel [3], [4], [5] and of matter. Additionally, there will also be a component $\eta_{\alpha\beta} M_{\mu\nu} M^{\mu\nu}$ where $M_{\mu\nu}$ is the tensor of the electromagnetic polarization of matter. This additional component has been obtained similarly as the component with contact interaction (spin) $\times$ (spin) in Einstein—Cartan’s theory. The new component may be treated as a contact interaction (electromagnetic moment) $\times$ (electromagnetic moment). The role of this component will be estimated and compared with the effects originated from the Einstein—Cartan theory.

We derive the second pair of Maxwell equations in terms of derivatives with respect to the connection with torsion. This will give us an additional internal current related to spin. From Bianchi’s identity, we get conservation laws of energy-momentum, angular momentum and charge.

2. THE KLEIN–KALUZA THEORY WITH TORSION

We introduce the electromagnetic bundle $P$ (for details see [6], [7]) with a natural metrization and a metrical connection $\omega^g$ invariant under a transformation of $U(1)$. The connection $\omega^g$ is not necessarily Riemannian. We also define a connection $\tilde{\omega}^{g\alpha}$ as metrical, but unnecessarily Riemannian on $E$. We assume that a bar above a symbol denoting connection, covariant derivation, curvature or other quantities, indicates that the quantity is defined on $E$. Whereas $\sim$ means a quantity depending on a Riemannian connection, e.g., $\tilde{\omega}^{g\alpha}$ means a Riemannian connection on $E$. Now we have $(E, g, \omega^g)$ a four-dimensional manifold with a metrical connection, metrical tensor $g$ with the signature $(- - - +)$, $(P, \gamma, \omega^g)$ a five-dimensional manifold with a metrical connection, metrical tensor $\gamma$ with the signature $(- - - - +)$. The connection $\omega^g$ is not necessarily Riemannian. We also define a connection $\tilde{\omega}^{g\alpha}$ as metrical, but unnecessarily Riemannian on $E$. We assume that a bar above a symbol denoting connection, covariant derivation, curvature or other quantities, indicates that the quantity is defined on $E$. Whereas $\sim$ means a quantity depending on a Riemannian connection, e.g., $\tilde{\omega}^{g\alpha}$ means a Riemannian connection on $E$. Now we have $(E, g, \omega^g)$ a four-dimensional manifold with a metrical connection, metrical tensor $g$ with the signature $(- - - +)$, $(P, \gamma, \omega^g)$ a five-dimensional manifold with a metrical connection, metrical tensor $\gamma$ with the signature $(- - - - +)$. The connection $\omega^g$ is not necessarily Riemannian. We also define a connection $\tilde{\omega}^{g\alpha}$ as metrical, but unnecessarily Riemannian on $E$. We assume that a bar above a symbol denoting connection, covariant derivation, curvature or other quantities, indicates that the quantity is defined on $E$. Whereas $\sim$ means a quantity depending on a Riemannian connection, e.g., $\tilde{\omega}^{g\alpha}$ means a Riemannian connection on $E$. Now we have $(E, g, \omega^g)$ a four-dimensional manifold with a metrical connection, metrical tensor $g$ with the signature $(- - - +)$, $(P, \gamma, \omega^g)$ a five-dimensional manifold with a metrical connection, metrical tensor $\gamma$ with the signature $(- - - - +)$.