ANALYSIS OF SUDDENLY STARTED LAMINAR FLOW IN THE ENTRANCE REGION OF A CIRCULAR TUBE

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Abstract
Suddenly started laminar flow in the entrance region of a circular tube, with constant inlet velocity, is investigated analytically by using integral momentum approach. A closed form solution to the integral momentum equation is obtained by the method of characteristics to determine boundary layer thickness, entrance length, velocity profile, and pressure gradient.

Nomenclature
\[ M(\xi, \tau, \eta) \] a function
\[ N(\xi, \tau, \eta) \] a function
\[ \rho \] pressure
\[ p^* \] \( p/\rho U^2 \), dimensionless pressure
\[ Q(\xi, \tau, \eta) \] a function
\[ R \] radius of the tube
\[ r \] radial distance
\[ Re \] \( 2RU/v \), Reynolds number
\[ t \] time
\[ U \] inlet velocity, constant for all time, uniform over the cross section
\[ u \] velocity in the boundary layer
\[ u^* \] \( u/U \), dimensionless velocity
\[ u_1 \] velocity in the inviscid core
\[ x \] axial distance
\[ y \] distance perpendicular to the axis of the tube
\[ y^* \] \( y/R \), dimensionless distance perpendicular to the axis
\[ \delta \] boundary layer thickness
\[ \delta^* \] displacement thickness
\[ \eta \] \( \delta/R \), dimensionless boundary layer thickness
\[ \theta \] momentum thickness
\[ \mu \] absolute viscosity of the fluid
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\[ \nu = \frac{\mu}{\rho}, \text{kinematic viscosity of the fluid} \]

\[ \xi = \frac{x}{(R \, Re)}, \text{dimensionless axial distance} \]

\[ \rho = \text{density of the fluid} \]

\[ \tau = \frac{t \, U}{(R \, Re)}, \text{dimensionless time} \]

\[ \tau_w = \text{wall shear stress} \]

§ 1. Introduction

In this work the unsteady laminar flow of a viscous, incompressible fluid in the entrance region of a circular tube is treated analytically. In particular, the problem of a fluid initially at rest and suddenly set in motion with inlet velocity constant for all time, under a constant head, is considered. Unsteady flow problems of this type have been neglected, probably because the time required to reach the steady state is short. However, a detailed analysis of such problems is desirable.

As a lead to the visualization of starting-flow development in the entrance region, a brief reference to the phenomenon of steady flow development is appropriate. A physical model that describes the steady state phenomenon consists of a boundary layer growing with axial distance until its thickness becomes equal to the tube radius, and a core of fluid in the central portion of the tube, outside the boundary layer, essentially undisturbed by the wall friction but accelerated to compensate for the flow retarded in the boundary layer. In this model the velocity distribution will vary gradually from some initial profile at the inlet to a fully developed (parabolic) profile at some location downstream. Correspondingly, the pressure gradient will undergo a gradual change over a certain length of the tube. The distance over which such a change takes

![Fig. 1. Boundary layer model for unsteady flow in the entrance region of a circular tube \((L_t)\) is the entrance length at time \(t\).]