EFFECT OF ROUGHNESS ON THE VELOCITY PROFILE OF A LAMINAR BOUNDARY LAYER *)

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Abstract
Flow past a rough wall is examined. Calculations are made to find the roughness-induced mean velocity which is expressed in an integral form in terms of the spectral density of the roughness and an influence function. Values of the influence function are tabulated using the known values of the modified Hankel functions of order \( \frac{1}{4} \) and their integrals. The first order change in lower critical Reynolds number due to the roughness-induced change in profile is calculated; the stability of the profile is increased due to the presence of roughness.

Nomenclature
- \( A_1, A_2, A_3 \) functions of \( \bar{u} \)
- \( c \) velocity of wave propagation in the boundary layer
- \( Z_1(\alpha, \beta, y) \) Fourier-Stieltjes transform of \( w_{1u} \)
- \( E(\bar{q}) \) two-dimensional spectral density of roughness distribution
- \( E, F, \psi, T \) various combinations of \( X_3 \) and \( X_4 \) defined by (67)
- \( F(\bar{q}) \) (isotropic) spectral density of roughness distribution
- \( G \) auxiliary function introduced to solve (53)
- \( h(x, z) \) roughness height

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§ 1. Statement of the problem
We consider steady incompressible flow past a semi-infinite rough wall whose roughness is randomly distributed in a given way. The average size and extent of the roughness is however limited by the assumption given below. Let us take the plane of the smooth wall as \( y = 0 \) and take \( h = h(x, z) \) as the height of a roughness, so that the equation of a rough surface is given by

\[
y = h(x, z). \tag{1}
\]

Let \(< >_{av}\) denote a spatial mean and suppose

\[
<h^{2}_{av} >^{\frac{1}{2}} = \bar{\varepsilon}. \tag{2}
\]

If \( \lambda \) is the Taylor microscale of the roughness distribution, we can interpret \( \lambda \) to be of the order of the distance between zeroes of