MASS TRANSFER IN THE LAMINAR BOUNDARY LAYER ALONG A FLAT PLATE CALCULATED BY MEANS OF THE INTEGRAL METHOD

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Summary
The approximation method propounded by von Kármán and Pohlhausen for hydrodynamic boundary layers has been applied to the calculation of the mass transfer in the laminar boundary layer along a flat plate without longitudinal pressure gradient. The velocity and concentration profiles are represented by polynomials of the 4th degree in terms of the distance from the wall. It is shown that for mass transfer this method leads to a rather complicated system of formulae. The calculations based upon this method are performed for several values of the Schmidt number $Sc$, viz. for $Sc = 0.1, 0.6, 1, 2, 5, 10, 50$ and $100$. The general trend of the results is in accordance with the exact theory. However, it appears that the numerical work involved is extensive and its accuracy disappointing. Furthermore, for Schmidt numbers lower than 1 the approximation method fails at certain negative values of the transfer parameter $B$.

List of symbols
- $A$ \text{ for } N = 0
- $B$ mass transfer parameter
- $C_f$ friction factor averaged over the length of the plate
- $c_n$ coefficient given by (24)
- $E$ \text{ for } $\eta = 0$
- $f$ reduced stream function
- $G$ function defined by (20)
- $L$ length of the plate
- $N$ \text{ for } $\lambda_D/(2-\lambda_D)$
- $Re$ Reynolds number
- $Sc$ Schmidt number
- $Sh$ Sherwood number
- $\zeta$ ratio between the thicknesses of the hydrodynamic and diffusion boundary layers
- $\beta$ reduced concentration profile
- $\delta_D$ thickness of diffusion boundary layer
§ 1. Introduction. In a previous paper\textsuperscript{1)} the boundary-layer equations for mass transfer in a laminar boundary layer along a flat plate without an exterior pressure gradient have been discussed. In the literature these equations have also been solved by means of approximation methods. One of the most famous methods is that propounded by von Kármán\textsuperscript{2)} and Pohlhausen\textsuperscript{3)}. In this method a simplified model of the boundary layers is introduced. It is assumed that the boundary layer, instead of merging asymptotically into the surrounding fluid, ends sharply at a certain distance from the wall. In this model we may distinguish between the thickness $\delta_H$ of the hydrodynamic boundary layer, the thickness $\delta_T$ of the thermal boundary layer and the thickness $\delta_D$ of the diffusion boundary layer. As a further simplification it is assumed that the tangential velocity, the temperature and the concentrations may be represented by simple functions of the distance $y$ from the wall. These functions are mostly represented by polynomials of the $n$th degree in $y$. We shall refer to the method in this form as the "$P_n$-method". The thicknesses of the boundary layers are calculated