ABSTRACT. Recent models in quantum cosmology make use of the concept of imaginary time. These models all conjecture a join between regions of imaginary time and regions of real time. We examine the model of James Hartle and Stephen Hawking to argue that the various ‘no-boundary’ attempts to interpret the transition from imaginary to real time in a logically consistent and physically significant way all fail. We believe this conclusion also applies to ‘quantum tunneling’ models, such as that proposed by Alexander Vilenkin. We conclude, therefore, that the notion of ‘emerging from imaginary time’ is incoherent. A consequence of this conclusion seems to be that the whole class of cosmological models appealing to imaginary time is thereby refuted.

1. INTRODUCTION

An intriguing but troublesome feature of recent models in quantum cosmology is their use of imaginary time. Resort to imaginary time is appealing because it seems to eliminate any initial spacetime singularity, thereby solving an embarrassing problem with the standard Friedmann models of Einstein’s general theory of relativity (GTR). But it is also troublesome, since it is unclear how to interpret imaginary time or what to make of the idea, found in ‘no-boundary’ models of quantum gravity, that real time ‘emerges’ from imaginary time. These models all conjecture a join between regions (or eras) of imaginary time and regions (or eras) of real time, so, for ease of reference, we shall call the latter problem the ‘join problem.’ To say that the ‘join problem’ is troublesome, however, is not to say that proponents of such models worry much about the meaning of imaginary time or about the transition from imaginary to real time. They don’t; but they should. For as we shall argue in this essay, various attempts to interpret the transition in a logically consistent and physically significant way all fail. Moreover, since there does not seem to be any way to solve the ‘join problem’ in a logically consistent and physically significant manner, we conclude that the notion of ‘emerging from imaginary time’ is incoherent. A consequence of this conclusion seems to be that the whole class of cosmological models appealing to imaginary time is thereby refuted.

A preliminary comment is needed. We think this conclusion applies to all ‘no-boundary’ models, but we think it applies as well to the ‘quantum
tunnelling' models, such as Alexander Vilenkin's,\textsuperscript{2} that make use the idea of imaginary time and that also conjecture some sort of transition between imaginary time and real time.\textsuperscript{3} However, to make our remarks specific, and to avoid having to discuss many models in so doing, we shall focus on a quantum model of the Universe proposed by James Hartle and Stephen Hawking (Hartle and Hawking, 1983) that has been refined over the last decade by Hawking in conjunction with various collaborators. For ease of reference, we refer to this model as the 'Hawking Universe' (HU).\textsuperscript{4} We begin with a description of the cosmological problem the HU is mainly intended to solve.

2. BACKGROUND

GTR is a theory of the overall structure and evolution of spacetime. It is a geometrical theory in which the structure and evolution of spacetime are codetermined by the mass-energy embedded in it. More precisely, the basic field equations of the theory equate the curvature of a spacetime to its mass-energy density and can be used to describe the temporal development of both. Few physicists doubt that GTR provides an accurate, realistic picture of the large-scale structure of the Universe and its evolution. The reasons are that non-static solutions to Einstein's field equations (the Friedmann ones, alluded to above) predict an expanding or contracting universe; and the best current evidence, from a variety of sources, indicates that the Universe is expanding in a manner described by these solutions. If we run the expansion backward, however, in search of its origin, we reach a singularity, a point of infinite spacetime curvature and infinite mass-energy density. Indeed, the equations of GTR, as solved for a Friedmann universe, entail a singularity at its origin where the field equations of GTR 'break down,' since they cannot be used to predict what will emerge from it (see, e.g., Munitz, 1986, 82–115; Isham, 1988, 388–392; Hawking, 1988, 46, 50–51, 122, 133; and Smith, 1988, 40–41). The upshot is that the 'standard Big-Bang (SBB) model' of the Universe, which assumes a Friedmann spacetime described by a Robertson–Walker metric, cannot describe its own beginning.

3. THE HAWKING UNIVERSE: AN OVERVIEW

Along with other physicists, Hawking finds this situation totally unsatisfactory (e.g., Hawking, 1988, 122–123, 148, 172–174; 1993, 89, 92). What to do? The approach of many physicists, including Hawking, has been to