ABSTRACT. Several accounts of logical truth are compared and shown to define distinct concepts. Nevertheless, conditions are given under which they happen to declare exactly the same sentences logically true. These conditions involve the variety of objects in the domain, the richness of the language, and the logical resources available. It is argued that the class of sentences declared logically true by each of the accounts depends on particularities of the actual world.

1. INTRODUCTION

There are a number of basic intuitions about logical truth. Wittgenstein (1961) writes “It is the peculiar mark of logical propositions that one can recognize that they are true from the symbol alone.”¹ Quine (1970) writes “A logically true ... sentence is a sentence whose truth ... is assured by its logical structure.”² Russell (1961) writes “A proposition such as ‘If Socrates is a man, and all men are mortal, then Socrates is mortal’, is true in virtue of its form alone ... The general truth of which it is an instance is purely formal, and belongs to logic.”³ And Russell (1919) writes “... logic ... aims at being true ... in all possible worlds ... and not only in this ... world.”⁴

It is certainly not obvious how to extract from these intuitions precise definitions demarcating logical truths from non-logical truths. As a matter of fact, a number of distinct accounts of logical truth have been discussed in the literature, each of which reflect some but usually not all of these basic intuitions about the nature of logical truth. Some of the definitions concern propositions. Others concern sentences or formulas. Some appeal to counterfactual possibilities. Others appeal only to the actual world.

Conceptually, many of the various definitions of logical truth that have been examined are distinguishable, implying that there are a number of distinct but at least initially plausible definitions. The diversity of competing accounts of such a fundamental notion suggests the need for a comparison of the various concepts, and a study of their relationship to basic intuitions.

Even the standard model-theoretic account of logical truth deriving from Tarski has come under criticism for defining a concept separable in principle from intuitive understandings of logical truth.⁵
The purpose of this paper is to study the connections between five different accounts of logical truth representative of those found in the literature. To make precise comparison possible, the definitions will be formulated for sentences rather than propositions, and only formal definitions will be examined. The accounts examined include the model-theoretic account, limited substitution, extended substitution, generalization, and permutation.

It will be shown that the five definitions are all conceptually distinct, that under adverse conditions any pair of them will differ as to which sentences are declared logically true. However, under favorable circumstances the various definitions will coincide. Thus, the core of the paper consists of a study of the conditions under which three of the five accounts agree: the model-theoretic account and the two substitutional accounts. This will depend on the size of the domain, on the variety of types of objects, on the linguistic resources available, and on the strength of the logical resources available.

In the final sections the relationship between the extension assigned to logical truth by various definitions and the structure of the actual world is examined. It is argued that many or all accounts of logical truth depend for their success on particularities of the actual world, implying that logical truth is more of a function of the actual world than has been thought.

2. DEFINITIONS OF LOGICAL TRUTH

Five definitions of logical truth are presented that have been examined, in one form or another, by logicians and philosophers of logic. The intent is not necessarily to positively attribute any of these approaches to particular historical or contemporary figures, and so the references given should be understood as suggestive of vague origin. In fact, for most of the definitions, there are many different variations. For example, substitutional definitions vary according to what types of substitutions are allowed, and the generalizational definition varies according to whether or not relativization is included.

Most of the definitions appeal to truth in the actual world. For the purposes of this paper the actual world will be represented by a first-order model, $W$, consisting of a particular domain of discourse (representing those objects that in fact exist) and an assignment of appropriate members of subsets of, and relations on the domain to linguistic elements of the appropriate types (representing the extensions that these names, predicate symbols and relation symbols happen to have).